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MODULE-1

Subject :- MECHANICS OF MATERIALS

SIMPLE STRESS AND STRAIN

Introduction, stress, strain, mechanical properties of materials, linear elasticity, Hooke's Law & Poission's ratio, Stress - strain relation - behaviour in tension for Mild steel & non ferrous metals. Extension / shortening of a bar, bars with cross-sections varying in steps, bars with continous varying cross sections (circular and rectangular), Elongation due to self weight. Principle of super-position [7 hours]

Introduction

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* Materials are classified into
  - elastic
  - plastic &
  - rigid materials

i). Elastic materials undergoes a deformation when subjected to an external loading such that the deformation disappears on the removal of the loading.

ii) Plastic material undergoes a continous deformation during the period of loading and the deformation is permanent and the material does not regain its original dimensions on removal of loading.

iii) Rigid material does not undergo any deformation when subjected to an external loading.

Stress

* The force of resistance offered by a body against the deformation is called stress.
* The external forces acting on the body is called the load.

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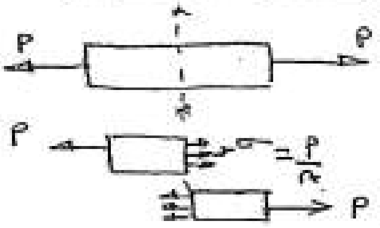
* The load is applied on the body while the stress is induced in the material of the body.

Types of stresses :-

ⓐ Tensile stress :- When a load is such that it tends to pull apart the particles of the material causing extension in the direction of application of the load, then the load is called the tensile load and the corresponding stress is tensile stress

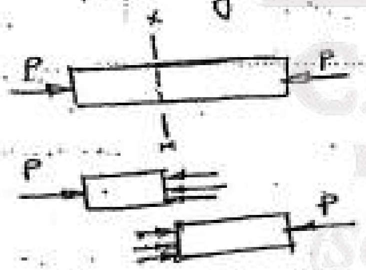


Here P is the applied load
A is the area of cross-section, then

$$\text{Tensile stress, } \sigma = \frac{\text{Resisting force } P}{\text{c/s area } A}$$

$$\sigma = \frac{P}{A} \text{ in } N/mm^2$$

ⓑ Compressive stress :- If a bar is subjected to pushing axial load as shown, a resistance is set up by any section such as x–x against a decrease in length. This resistance is called compressive stress.



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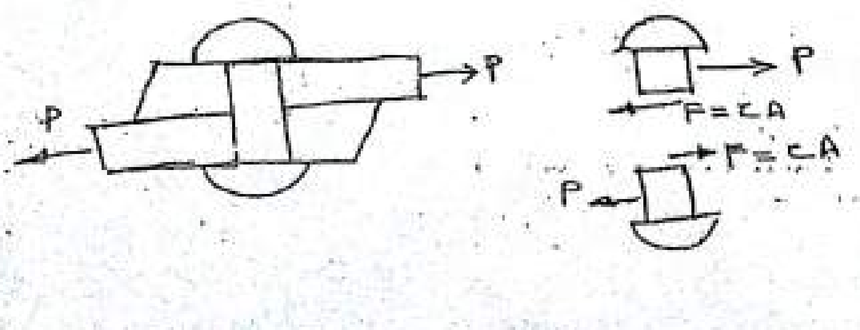
The intensity of compressive resistance or stress is given by $\sigma = \frac{P}{A}$.

ⓒ Shear stress



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A body is said to be subjected to shear stress, when two equal and opposite forces act tangential on a plane. stress induced in the plane of a section is known as shear stress.

Shear stress $\tau = \frac{\text{shear force}}{\text{c/s area}} = \frac{F_t}{A}$

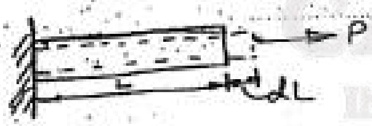
Strain

* Change in dimensions of a structural member subjected to forces is known as deformation and which may be measured as increment or decrement in the dimensions.
* The ratio of change in length to the original length of a member is called strain.

strain, $\epsilon = \frac{\text{change in length}}{\text{Original length}} = \frac{dL}{L}$

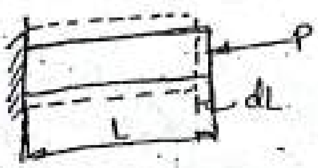
Tensile strain

* When the resistance offered by a section of a member is against an increase in length, the section is said to offer a tensile stress. The corresponding strain is called tensile strain



Tensile strain, $\epsilon = \frac{\text{Increase in length}}{\text{original length}} = \frac{dL}{L}$

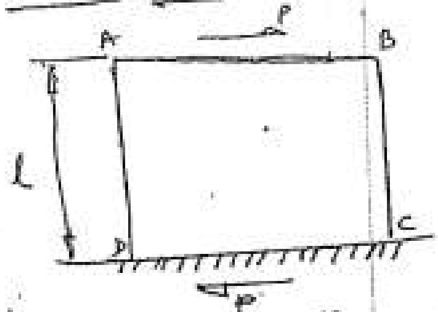
Compressive stress

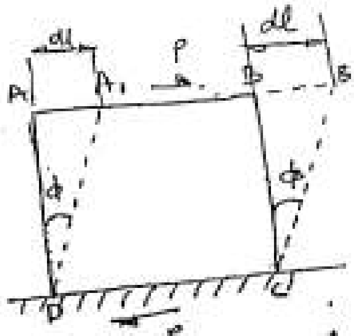


Compressive strain, $\epsilon = \frac{\text{decrease in length}}{\text{Original length}}$
$= \frac{dL}{L}$

source: diginotes.in

Shear strain





* The above figure shows a rectangular block subjected to shear forces P on its top and bottom faces. When the block does not fail in shear, a shear deformation occurs as shown.
* If the bottom face of the block is fixed, it can be realised that the block has deformed to the position $A_1 B_1 C D$. Or we can say, the face ABCD has been distorted to the position $A_1 B_1 C D$ through an angle, $B C B_1 = \phi$
* Let the horizontal displacement of the upper face of the block be dl. Let the height of the block be l

Now, shear-strain $= \frac{dl}{l} = \frac{\text{transverse displacement}}{\text{distance from the lower face}}$

Here shear strain $= \phi \simeq \tan\phi = \frac{dl}{l}$.

Stress-strain diagram :-

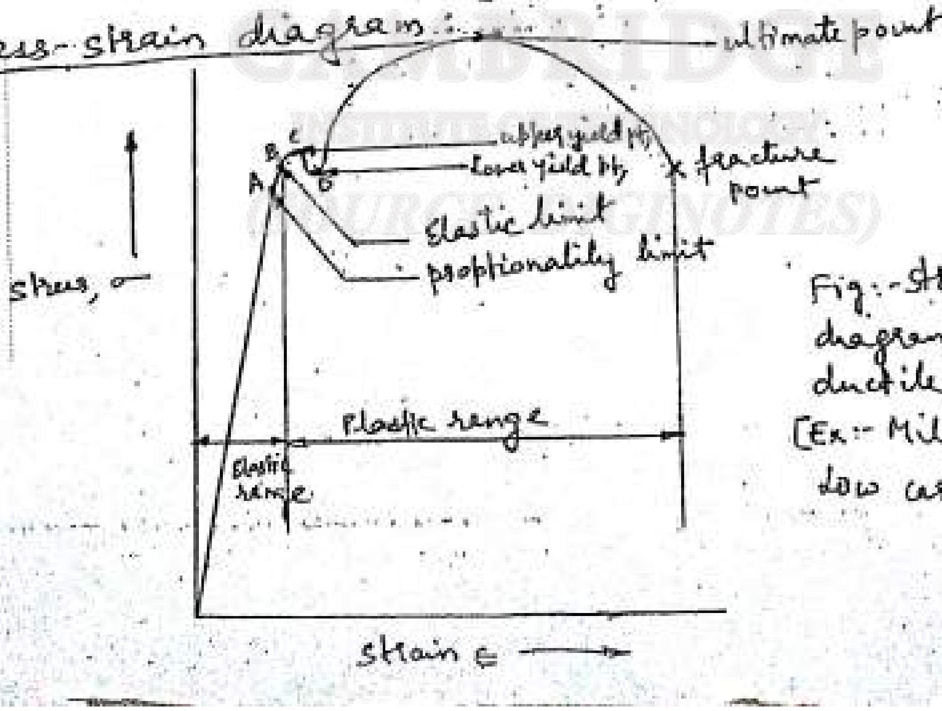


Fig:- Stress-strain diagram for ductile material [Ex:- Mild steel or low carbon steel]

source: diginotes.in

* Uniaxial tensile test is conducted on a standard specimen made of ductile material or brittle material to obtain information regarding the behaviour of a given material under gradually increasing stress and strain condition.
* A standard specimen is subjected to gradually increasing axial load W and the values of loads corresponding at a regular intervals are noted.

i) Elastic range

→ In this range the material is elastic in nature.
→ Elasticy is defined as "the property of the material by the virtue of which deformations caused by stress disappear on removal of load."

ii) Proptional limit

→ It is the maximum stress level upto which the stress is directly proptional to strain.
→ For some materials elastic limit is slightly above the proptional limit.

iii) Yield stress (σ_y)

→ Yield stress is the value of stress at which the material continues to deform at constant load condition.
→ Two distinct points shown on the curve in above figure are the highest stress preceeding extensive strain known as upper yield point and relatively constant runout value known as lower yield point.

iv) Plastic range

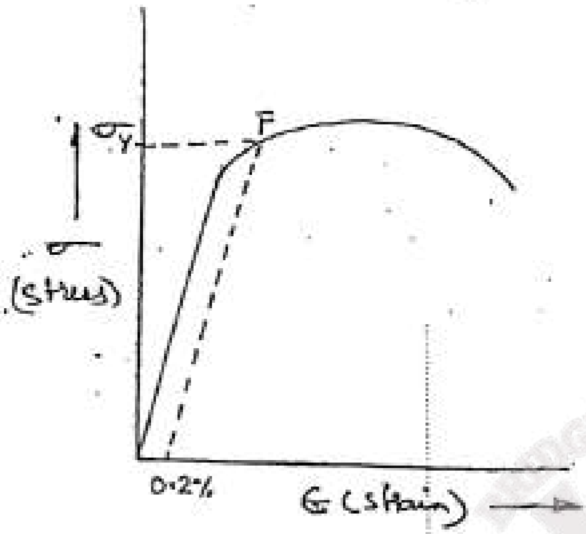
→ After elastic limit the specimen undergoes deformation which cannot be regained with removal of load. This deformation is known as plastic deformation.

v) Ultimate stress (σ_u)

→ It is the maximum stress induced in the specimen and it occurs in the plastic region.

source: diginotes.in

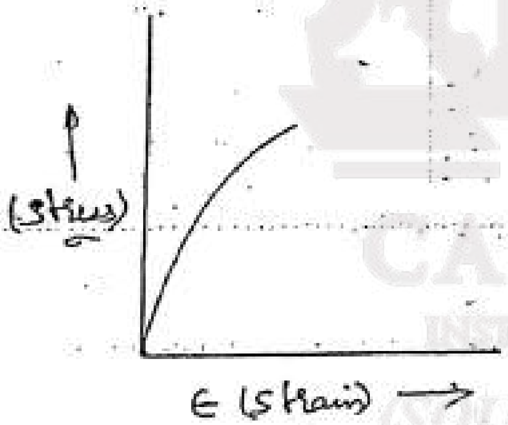
Stress strain diagram for high strength steel



* In high strength (or high carbon steel) there is no clear-cut yield point.
* Necking takes place at ultimate stress and eventually the breaking point is lower than the ultimate point.

* The stress σ_Y at which if unloading is made there will be 0.2% (or 0.002) strain, is known as 0.2% proof stress and this point is treated as yield point for all pratical purposes.

Stress - strain relation diagram in brittle material



* In brittle materials such as cast iron, glass, wood etc, there is no appreciable change in the rate of strain. There is no yield point and no necking takes place.
* Ultimate point and breaking point are one and the same. The strain at failure is very small.

Hooke's Law :-

* It is observed that when a material is loaded such that the intensity of stress is within a elastic limit, the ratio of the intensity of stress to the corresponding strain is a constant which is charateristic of the material.

source: diginotes.in

i.e.,

stress (σ) $\propto$ strain (ϵ) within elastic limit

or $\frac{\sigma}{\epsilon} = E$,

Where E is a constant known as modulus of elasticity or Young's modulus.

* In case of shear loading, the ratio of shear stress and the corresponding shear strain is found to be constant when the shear deformation is within a certain limit i.e,

shear stress (τ) $\propto$ shear strain (ϵ_s) within elastic limit

or $\frac{\tau}{\epsilon_s} = G$

Where G is a constant known as modulus of rigidity or shear modulus.

Percentage elongation and percentage reduction in area :-

* Percentage elongation and percentage reduction in area are the two terms used to measure the ductility of material.

a) Percentage Elongation :- It is defined as the ratio of the final extension at rupture to original length expressed as percentage. Thus,

$$\text{\%age elongation} = \frac{L_1' - L}{L} \times 100 = \frac{L_f - L_0}{L_0} \times 100$$

where L_0 (or L) is original length and L_1' (or L_f) is final length.

b) Percentage Reduction in area :- It is defined as the ratio of maximum changes in the cross-sectional area to original area. Thus

source: diginotes.in

% age reduction in area $= \frac{A_0 - A_f}{A_0} \times 100$

where A_0 is original cross-sectional area and A_f is the final cross-sectional area.

Problem 1) The steel specimen of 12.5mm diameter and 150mm gauge length is subjected to a tensile test. It is observed that the load at yield point is 43 KN and the maximum load is 60 KN. A load of 16.4 KN is required to cause an elastic extension of 0.1 mm. Final length of specimen is 190mm and the diameter of neck after fracture is 8 mm. Determine,
i) yield stress ii) ultimate stress iii) Young's modulus
iv) % age increase in length v) % age elongation reduction in area.

Soln.

Given

Original diameter $d_0 = 12.5\,mm$

Original length $l_0 = 150\,mm$

Load at yield point $P_y = 43\,KN = 43 \times 10^3\,N$

Maximum (or ultimate) load, $P_u = 60\,KN = 60 \times 10^3\,N$

Load applied $P = 16.4\,KN = 16.4 \times 10^3\,N$

change in length $dl = 0.1\,mm$

final length $l_f = 190\,mm$

final diameter $d_f = 8\,mm$.

Original c/s area $A_0 = \frac{\pi}{4} d_0^2 = \frac{\pi}{4} \times (12.5)^2 = 122.72\,mm^2$

i) Yield stress (σ_y)

$$\sigma_y = \frac{\text{load at yield point}}{\text{Original c/s. area}} = \frac{P_y}{A_0} = \frac{43 \times 10^3}{122.72}$$

$$\sigma_y = 350.39\,N/mm^2$$

source: diginotes.in

ii) ultimate stress (σ_u)

$$\sigma_u = \frac{\text{load at ultimate point}}{\text{original c/s area}} = \frac{P_u}{A_o}$$

$$\therefore \sigma_u = \frac{60 \times 10^3}{122.72} = 488.92 \text{ N/mm}^2$$

iii) Young's Modulus (E)

Now $E = \frac{\sigma}{\epsilon}$ within elastic limit

$$\text{stress } \sigma = \frac{P}{A_o} = \frac{16.4 \times 10^3}{122.72} = 133.64 \text{ N/mm}^2$$

$$\text{strain } \epsilon = \frac{dl}{L_o} = \frac{0.1}{150} = 6.67 \times 10^{-4}$$

$$\therefore E = \frac{133.64}{6.67 \times 10^{-4}} = 2.004 \times 10^5 \text{ N/mm}^2$$

iv) %age elongation (or % increase in length)

$$\frac{l_f - l_o}{l_o} \times 100 = \frac{190 - 150}{150} \times 100 = 26.67\%$$

v) %age reduction in area

$$\frac{A_o - A_f}{A_o} \times 100 = \frac{\frac{\pi}{4} d_o^2 - \frac{\pi}{4} d_f^2}{\frac{\pi}{4} d_o^2} \times 100$$

$$= \frac{d_o^2 - d_f^2}{d_o^2} \times 100 = \frac{12.5^2 - 8^2}{12.5^2} \times 100$$

$$= 59.04\%$$

source: diginotes.in

June/July 08 - 10 Marks

1b2) For the laboratory tested speciman the following data was obtained:

i) Diameter of the speciman = 25 mm
ii) Length of the speciman = 300 mm
iii) Extension under the load of 15 KN = 0.045 mm
iv) Load at yield point = 127.65 KN
v) Maximum load = 208.60 KN
vi) Length of the speciman after failure = 375 mm
vii) Neck diameter = 17.75 mm.

Determine i) Young's modulus. ii) Yield point stress. iii) ultimate stress iv) %age elongation v) %age reduction in area.

Sol^n:

Given

Original diameter of speciman, $d_0 = 25$ mm
Original length $l_0 = 300$ mm
Load applied (within elastic limit) $P = 15$ KN $= 15 \times 10^3$ N
Extension (or change in length) $dl = 0.045$ mm
Load at yield point $P_y = 127.65$ KN $= 127.65 \times 10^3$ N
Maximum load (or ultimate load) $P_u = 208.60$ KN $= 208.60 \times 10^3$ N
final length $L_f = 375$ mm.
final diameter $d_f = 17.75$ mm

i) Young's modulus (E)

$$E = \frac{\sigma}{\epsilon} \quad \text{within elastic limit}$$

where stress $\sigma = \frac{P}{A_0}$

Original c/s area $A_0 = \frac{\pi}{4} d_0^2 = \frac{\pi}{4} \times 25^2 = 490.87$ mm^2

$$\therefore \sigma = \frac{15 \times 10^3}{490.87} = 30.58 \text{ N/mm}^2$$

$$\text{strain} = \frac{dl}{L_0} = \frac{0.045}{300} = 1.5 \times 10^{-4}$$

CAMBRIDGE INSTITUTE OF TECHNOLOGY (SOURCE DIGINOTES)

$$\therefore E = \frac{30.58}{1.5 \times 10^{-4}} = 2.04 \times 10^5 \ N/mm^2$$

(ii) Yield point stress (σ_Y)

$$\sigma_Y = \frac{P_{ye}}{A_0.} = \frac{127.65 \times 10^3}{490.87}$$

$$\therefore \sigma_Y = 260.05 \ N/mm^2$$

(iii) Ultimate stress (σ_u)

$$\sigma_u = \frac{P_u}{A_0} = \frac{208.60 \times 10^3}{490.87}$$

$$\therefore \sigma_u = 424.96 \ N/mm^2$$

(iv) %age elongation

$$\frac{L_f - L_0}{L_0} \times 100 = \frac{375 - 300}{300} \times 100 = 25\%$$

(v) %age reduction in Area

$$\frac{A_0 - A_f}{A_0} \times 100 = \frac{\frac{\pi}{4} d_0^2 - \frac{\pi}{4} d_f^2}{\frac{\pi}{4} d_0^2} \times 100$$

$$\therefore \frac{d_0^2 - d_f^2}{d_0^2} \times 100 = \frac{25^2 - 17.75^2}{25^2} \times 100$$

$$= 49.59\%$$

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source: diginotes.in

Dec 08 (old scheme) - 10 Marks

Pb 3) The following observations were made in a tension test on mild steel rod of diameter 10mm & length 200 mm.

Extension under a load of 10 KN = 0.12 mm,

Maximum load = 26 KN,

load beyond which stress-strain curve was not proportional = 11 KN,

Length at failure = 261.5 mm,

Diameter at failure = 5.7 mm.

Find the limit of proportionality, Young's modulus, percentage elongation of length and percentage contraction of area at failure.

Sol^n Given

Original diameter $d_o = 10\,mm$

Original length $l_o = 200\,mm$

Under elastic limit { Load $P = 10\,KN = 10 \times 10^3\,N$
Extension $dl = 0.12\,mm$

Maximum (ultimate) load $P_u = 26\,KN = 26 \times 10^3\,N$

load beyond which stress-strain curve was not proportional (i.e. load at yield point) } $P_y = 11\,KN = 11 \times 10^3\,N$

length at failure $l_f = 261.5\,mm$

Diameter at failure $d_f = 5.7\,mm$.

(i) Limit of proportionality (yield stress) σ_y :-

$$\sigma_y = \frac{P_y}{A_o}$$

where original c/s area $A_o = \frac{\pi}{4} d_o^2 = \frac{\pi}{4} \times 10^2$

$\therefore A_o = 78.54\,mm^2$

$$\therefore \sigma_y = \frac{11 \times 10^3}{78.54} = 140.06\,N/mm^2$$

source: diginotes.in

(ii) Young's modulus E

$$E = \frac{\sigma}{\epsilon} \quad \text{within elastic limit}$$

$$\text{stress } \sigma = \frac{P}{A_0} = \frac{10 \times 10^3}{78.54} = 127.32 \text{ N/mm}^2$$

$$\text{strain } \epsilon = \frac{dL}{L_0} = \frac{0.12}{200} = 6 \times 10^{-4}.$$

$$\therefore \text{Young's modulus } E = \frac{\sigma}{\epsilon} = \frac{127.32}{6 \times 10^{-4}} = 2.12 \times 10^5 \text{ N/}$$

(iii) Percentage elongation of length.

$$\frac{l_f - l_0}{l_0} \times 100 = \frac{261.5 - 200}{200} \times 100$$

$$= 30.75\%$$

(iv) Percentage contraction (reduction) in area.

$$\frac{A_0 - A_f}{A_0} \times 100 = \frac{\frac{\pi}{4} d_0^2 - \frac{\pi}{4} d_f^2}{\frac{\pi}{4} d_0^2} \times 100$$

or

$$\frac{d_0^2 - d_f^2}{d_0^2} \times 100 = \frac{10^2 - 5.7^2}{10^2} \times 100$$

$$= 67.51\%.$$

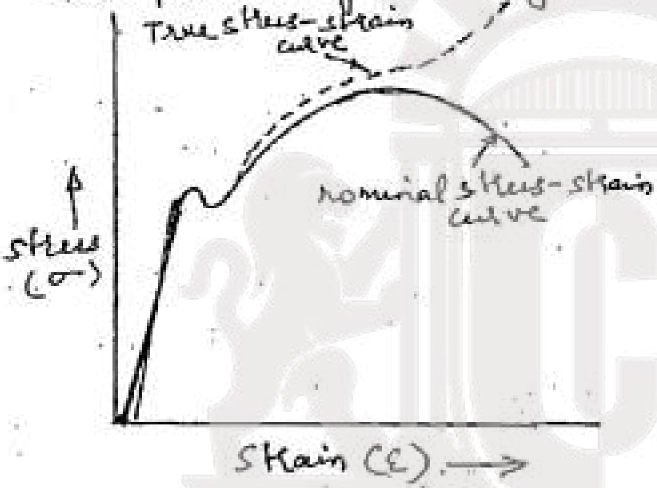
source: diginotes.in

True stress and true strain

* Engineering stress or nominal stress is defined as

$$\text{Engg stress or nominal stress} \quad \sigma = \frac{\text{load}}{\text{original c/s area}}$$

* However a ductile specimen loaded beyond yeild strength undergoes appreciable change in its dimensions. A rapid reduction in cross-sectional area of the specimen is observed at its critical section during the process of necking.



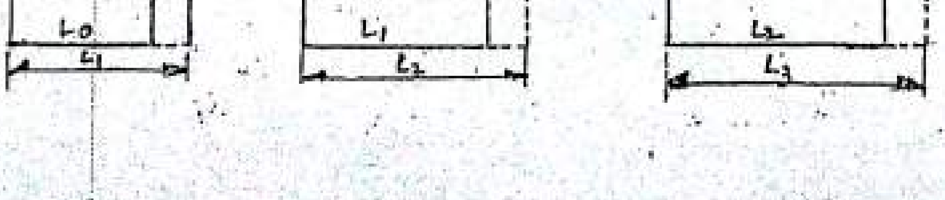
We define

$$\text{True stress} = \frac{\text{Load}}{\text{Actual c/s area}}$$

* Engineering strain or nominal strain is defined as

$$\text{Engg strain or nominal strain} \quad \varepsilon = \frac{\text{change in length}}{\text{Original length}}$$

* Engineering strain calculated based on original length is not a realistic measure, where large strains are involved. In such cases it is appropriate to use true strain which is defined as the ratio between change in gauge length and instantaneous gauge length.



true strain $\epsilon_t = \Sigma \frac{L_1 - L_0}{L_0} + \frac{L_2 - L_1}{L_1} + \frac{L_3 - L_2}{L_2} + \cdots$

$$\epsilon_t = \int_{L_0}^{L_i} \frac{dL}{L_i} = \log_e \frac{L_i}{L_0}.$$

where L_i = length of the specimen at any instant of test ($i = 1, 2, 3 \ldots$).

Properties of Engineering materials

a) stiffness (K) :-

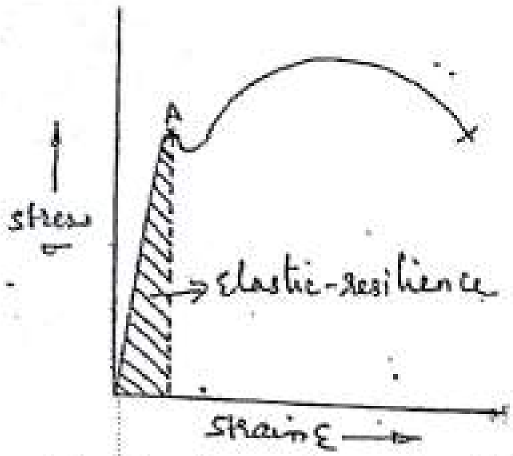
* stiffness is defined as the resistance offered by the material to elastic deformation.
* Material having high stiffness show less deformation under load.
* The modulus of elasticity or the Young's modulus (E), itself is the measure of stiffness of the material. Materials having high value of E show higher stiffness.

stiffness, $K = \frac{\text{load}}{\text{deflection}} = \frac{P}{\Delta L}$

b) Resilence

* Resilence is the ability of a material to absorb energy when it is elastically deformed and then upon unloading, to have this energy recovered.
* So as long as the body remains loaded, it contains stored energy within itself, which is called strain energy. As soon as the load is removed, the stored energy is given back, exactly as observed in spring

source: diginotes.in



* The strain energy stored by the material per unit volume at the elastic limit is known as the modulus of resilience.
* This property gives the capacity of the material to withstand shocks and vibration.

* It is given by the relation

$$U_r = \frac{\sigma_s^2}{2E}$$

where U_r = modulus of resilience

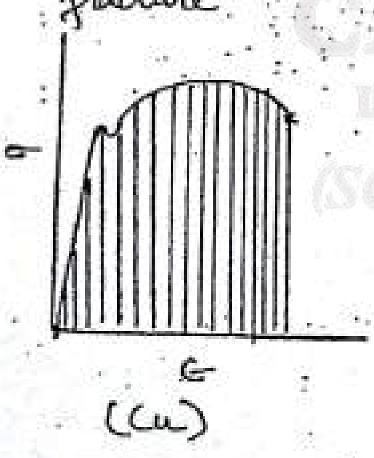
σ_s = stress at a point A and

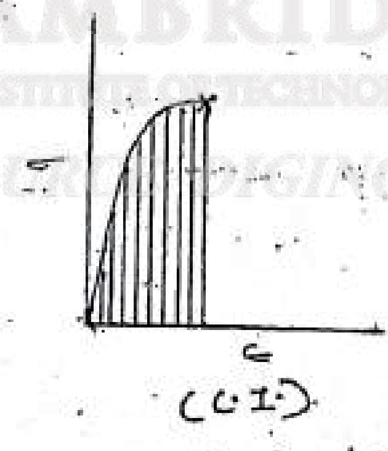
E = Young's modulus.

* Materials having high elastic limit have high resilience. materials having high resilience are used for springs.

c) Toughness

* Toughness is the ~~ability~~ ability of the material to absorb energy during plastic deformation. Toughness refers to the ability of a material to withstand bending or the application of shear stress without fracture.





* By definition copper is extremely tough while cast Iron is not.

* Area under the stress-strain diagram represents toughness per unit volume of material and is known as modulus of toughness.

source: diginotes.in

* It is given by the relation,

$$T = \left(\frac{\sigma_y + \sigma_u}{2}\right)\epsilon_f$$

where, σ_y = yield stress
σ_u = ultimate stress and
ϵ_f = strain at fracture point.

* satisfactory performance of certain parts such as drilling equipment, automotive equipment etc, depends on their toughness.

d) Hardness

* Hardness is the resistance of a material to plastic deformation usually by indentation. However the term may refer to resistance to ~~stat~~ scratching, abrasion or cutting.
* Tests such as Brinell, Rockwell, Vickers etc are generally employed to measure hardness.
* C.I. and ~~k~~ hardened (or high carbon) steel are very hard materials, brass is considered to be of intermediate hardness, whereas metals such as copper, silver and gold are soft materials and posses very low hardness. Diamond is the hardest material known.

e) Impact strength

* Impact strength is a complex charateristic which takes into account both toughness and strength of a material. "The capacity of a material to resist or absorb energy before it fractures is called impact strength."
* Impact strength is sensitive to rate of loading and to temperatures as well as stress raisers or stress concentrators such as notches, grooves, keyway etc
* Ductile materials posses higher impact strength than brittle materials.

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Factor of safety :-

* It is obvious that one cannot take risk of loading a member to its ultimate strength in practice. The maximum stress to which the material of a member is subjected in practice is called 'working stress'.

* This value should be well within the elastic limit in elastic design method and should be well within the ultimate strength for ultimate load design method. To avoid permanent deformation in the member working stress is kept less within than elastic limit.

* The ratio of yield stress to working stress is called Factor of ~~sagfary~~ safety.

$$\therefore \text{FOS} = \frac{\text{Yield stress}}{\text{Working stress}} \quad \text{for ductile materials.}$$

$$\text{and FOS} = \frac{\text{Ultimate stress}}{\text{working stress}} \quad \text{for brittle materials}$$

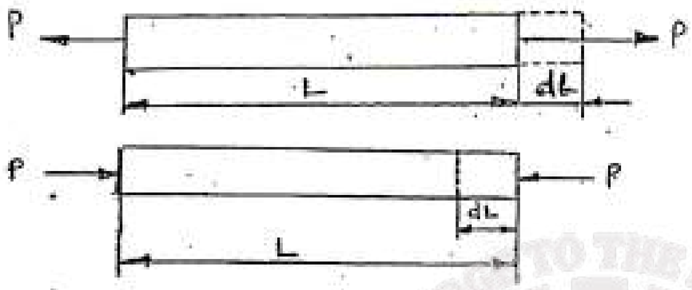
* FOS for specific applications is fixed based on uncertainities like:
  - (i) Unexpected load (gradually or suddenly acting)
  - (ii) Manufacturing defects such as blow holes in castings, fabrication errors. etc.
  - (iii) Environmental effects such as corrosion and wear
  - (iv) Temperature effects
  - (v) Uncertainities related to strength of materials.

* Factor of safety for various materials depends up on their reliability. The following values are commonly taken in ~~practice~~ practice.
  1) For steel ⇒ upto 1.85
  2) For concrete ⇒ upto 3
  3) For timber ⇒ 4 to 6.

source: diginotes.in

Extension / shortening of a bar

* A body subjected to axial load undergoes change in its linear dimension, known as deformation.



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$$\text{Stress } \sigma = \frac{\text{load}}{\text{Original c/s area}} = \frac{P}{A}$$

$$\text{Strain } \epsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{dL}{L}$$

From Hooke's law, we have

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\epsilon} = \frac{P/A}{dL/L}$$

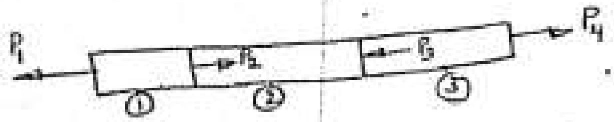
or $$E \frac{dL}{L} = \frac{P}{A}$$

$$\boxed{dL = \frac{PL}{AE}} \quad \text{or} \quad \boxed{dL = \frac{\sigma L}{E}}$$

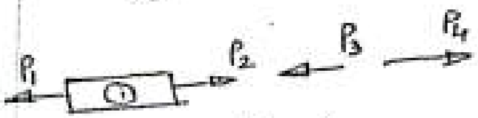
Principle of super-position :-

* Most of the machine members are subjected to the combination of various loads. The combined effect of load in the form of stress and strains may be found by using the principle of superposition.
* The principle of superposition state that, "Total effect of several loads applied on the body is the sum of effects of individual loads applied separately."

source: diginotes.in

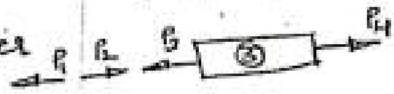


For 1st member



For 2nd member

For 3rd member



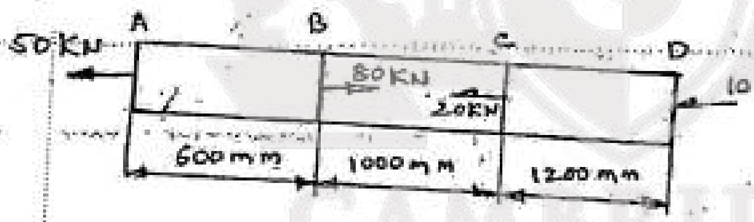
Total deformation $dL = dL_1 + dL_2 + dL_3$

The principle can be applied only when

1) The effects such as stress and strain are directly proportional to the loads which produce them.
2) The strain produced are small.

May 2010 - 10 Marks

Pb4) A brass bar having a cross-sectional area of $1000 mm^2$ is subjected to axial forces as shown. Determine the total elongation of the bar, if $E = 105 GPa$.



Soln.

Given, c/s Area $A = 1000 mm^2 = A_1 = A_2 = A_3$

Young's modulus $E = 105 GPa = 105 \times 10^9 \frac{N}{m^2}$

$= 105 \times 10^9 \frac{N}{(1 \times 10^3)^2 mm^2}$

$= \frac{105 \times 10^9}{1 \times 10^6} \frac{N}{mm^2} = 105 \times 10^{9-6} \frac{N}{mm^2}$

$= 105 \times 10^3 N/mm^2$

or $E = 1.05 \times 10^5 N/mm^2$

source: diginotes.in

1) A rod 1500 mm length & diameter 20 mm is subjected to axial load of 20 kN. If the Young's Modulus of the material is $2\times10^5\ N/mm^2$, find.
i) Stress (ii) Strain (iii) Deformation of Rod

Sol: Given data.

Length $= L = 1500$ mm
Diameter $= d = 20$ mm
Load $= P = 20$ kN
Young's Modulus $= E = 2\times10^5\ N/mm^2$

To Find
i) Stress $= \sigma = \frac{Load}{Area} = \frac{P}{A}$
ii) Strain $= \varepsilon = \frac{\delta L}{L}$
(or) $\varepsilon = \frac{\sigma}{E}$
iii) Deformation of rod $= \delta L = \frac{PL}{AE}$

$$\Rightarrow \sigma = \frac{P}{A} = \frac{P}{\left(\frac{\pi d^2}{4}\right)} = \frac{4P}{\pi d^2}$$

$$= \frac{4\times 20\times10^3}{\pi\times(20)^2} = 63.66\ N/mm^2$$

$$\therefore \boxed{\sigma = 63.66\ N/mm^2}$$

ii) $\varepsilon = \frac{\sigma}{E} = \frac{63.66}{2\times10^5} = 3.183\times10^{-4}$

$$\therefore \boxed{\varepsilon = 31.83\times10^{-5}}$$

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source: diginotes.in

iii) Deformation $\delta L = \frac{PL}{AE} = \varepsilon L$

$= 31.83 \times 10^{-5} \times 1500$

$\delta L = 0.477$ mm

2) Find the minimum diameter of steel wire which is used to raise a load of 4000N. If the stress in the wire is not to exceed 95 MN/m^2.

Sol: Given data

Load = P = 4000 N

Stress = σ = 95×10^6 N/m^2

To find

Minimum diameter = d = ?

We know that $\sigma = \frac{P}{A} = \frac{P}{\left(\frac{\pi d^2}{4}\right)}$

$\therefore 95 \times 10^6 = \frac{4000 \times 4}{\pi \times d^2}$

$d^2 = \frac{4000 \times 4}{\pi \times 95 \times 10^6} = 5.361 \times 10^{-5}$

$\therefore$ $d = 7.32 \times 10^{-3}$ m

$d = 7.32$ mm

source: diginotes.in

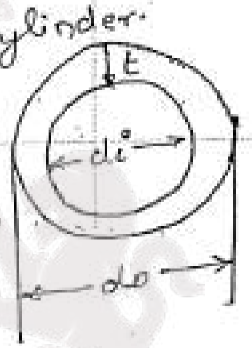
Q) A hollow cast iron cylindrical of 4m long, 300mm outerdiameter & thickness of metal is 500mm, is subjected to a central load on the top. The stress produced is 75 N/mm². Take Young's Modulus for cast iron 1.5×10^5 N/mm². Find

i) Magnitude of load

ii) Longitudinal strain

iii) change in length.

Sol: Let d_o = Outer diameter of hollow cylinder.



∴ d_o = 300 mm

t = thickness of cylinder

t = 50.0 mm

L = length

= 4m = 4000 mm

σ = Stress = 75 N/mm²

E = Young's Modulus = 1.5×10^5 N/mm²

To Find

i) Load = $P = \sigma \times A$

ii) Longitudinal strain = $\frac{\delta L}{L}$

iii) change in length = deformation.

Area of hollow cylinder = $\frac{\pi(d_o^2 - d_i^2)}{4}$

d_i : Inner diameter of hollow cylinder.

$\frac{d_o - d_i}{2} = t$

$d_i = -2t + d_o = 300 - 2 \times 50$

$d_i = 200$ mm

source: diginotes.in

$\therefore \quad P = \frac{\sigma \times 4}{\pi (d_o^2 - d_i^2)} = 75$

$\therefore \quad P = \sigma \times \frac{\pi (d_o^2 - d_i^2)}{4}$

$= \frac{75 \times \pi \times (300^2 - 200^2)}{4}$

$\boxed{P = 2.95 \times 10^6 \, N}$

ii) $\varepsilon = \frac{\sigma}{E} = \frac{75}{1.5 \times 10^5} = 50 \times 10^{-5}$

Longitudinal strain $= \varepsilon = 50 \times 10^{-5}$

iii) Change in length $= \delta L = L \times \varepsilon$

$= 50 \times E^{-5} \times 4000$

$= 2 \, mm.$

Q) A circular rod of diameter 20mm & 500mm long is subjected to a tensile force 45KN. The modulus of elasticity for steel may be taken as 200 KN/mm². Find stress, strain & elongation of the bar due to applied load

Sol: Given data. Load $= P = 45 \, KN$

$E = 200 \, KN/mm^2$

$d = 20 \, mm$

$L = 500 \, mm$

To find:

i) Stress $= \sigma = \frac{P}{A}$ (ii) $\varepsilon = \frac{\sigma}{E}$

iii) elongation $= \delta L = \frac{PL}{AE}$

source: diginotes.in

$$\therefore \sigma = \frac{4 \times P}{\pi d^2} = \frac{4 \times 45 \times 10^3}{\pi (20)^2}$$

$$= 143.24 \ N/mm^2$$

$$\varepsilon = \frac{\sigma}{E} = \frac{143.24}{200 \times 10^3} = 0.0007162$$

$$\delta L = \frac{PL}{AE} = \frac{45 \times 10^3 \times 500 \times 4}{\pi \times 20^2 \times 200 \times 10^3}$$

$$\text{(or)} \quad \delta L = \varepsilon L = 0.0007162 \times 500$$

$$= 0.358 \ mm$$

Q A specimen of steel 25mm diameter with a gauge length [original length) of 200 mm is tested to destruction. It has an extension of 0.16 mm under a load of 80 kN & the load @ elastic limit is 160 kN. The maximum load is 180 kN. The total extension @ fracture is 56mm & diameter @ neck is 18mm. Find

i) Stress @ elastic limit

ii) Youngs Modulus

iii) Percentage of elongation

iv) Percentage of reduction in area

v) Ultimate tensile stress

If the load @ failure is 150 kN, then find Nominal & True breaking stress.

source: diginotes.in

Sol: Given data

$d = 25$ mm

$l = 200$ mm

$\delta L = 0.16$ mm under load of 80 kN

$P = 160$ kN load @ elastic limit

$P_{max} = 180$ kN: Maximum load

$L' - L = 56$ mm = Total extension

$d_f = 18$ mm = dia @ neck or failure.

$P_f = 150$ kN = Failure load

To find

i) Stress @ elastic limit

$$\sigma_{(elastic\ limit)} = \frac{\text{load @ elastic limit}}{\text{original c/s Area}} = \frac{P}{A}$$

$$A = \frac{\pi d^2}{4} = 490.874 \text{ mm}^2$$

$$\therefore \sigma = \frac{160 \times 10^3}{490.874} = 325.949 \text{ N/mm}^2$$

ii) Young's Modulus $E = \frac{\text{Stress}}{\text{Strain}}$ [within the elastic limit]

$$= \frac{(P/A)}{(\delta L/L)} = \frac{PL}{A \cdot \delta L}$$

$$= \frac{80 \times 10^3 \times 200}{490.874 \times 0.16}$$

$$= 203.718 \text{ kN/mm}^2$$

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iii) Percentage of elongation = $\frac{\text{Final extension}}{\text{original length}} \times 100$

$$= \frac{L' - L}{L} \times 100$$

$$= \frac{56}{200} \times 100 = \underline{28\,\%}$$

iv) Percentage of reduction in Area

$$= \frac{\text{Initial area} - \text{Final area}}{\text{Initial area}} \times 100$$

$$= \frac{\left(\frac{\pi \times 25^2}{4}\right) - \left(\frac{\pi \times 18^2}{4}\right)}{\left(\frac{\pi \times 25^2}{4}\right)} \times 100$$

$$= \underline{48.16}$$

v) Ultimate tensile stress = $\frac{\text{Ultimate load}}{\text{Area}}$

$$\sigma_{ulti} = \frac{P_{max}}{A} = \frac{180 \times 10^3}{490.874}$$

$$= \underline{366.693\ N/mm^2}$$

vi) Nominal breaking stress = $\frac{\text{Load @ failure}}{\text{Original c/s Area}} = \frac{P_f}{A}$

$$= \frac{150 \times 10^3}{490.874} = \underline{305.58\ N/mm^2}$$

vii) True breaking stress = $\frac{\text{failure load}}{\text{Actual c/s Area}} = \frac{P_f}{A_0}$

$$= \frac{150 \times 10^3 \times 4}{\pi \times 18^2} = 589.463\ N/mm^2$$

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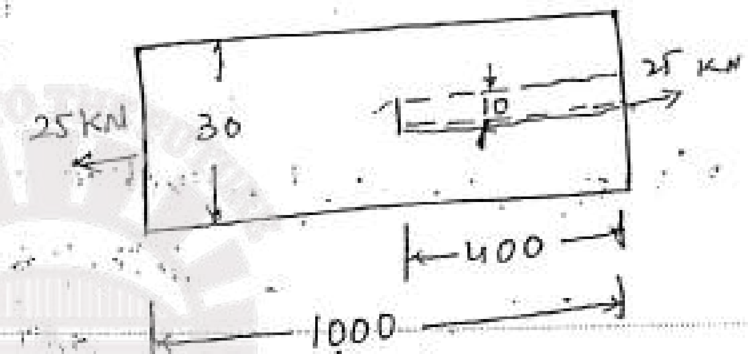
5) A bar of length 1000 mm & dia 30 mm is centrally bored for 400 mm, the bore diameter being 10 mm as shown in fig 2.23. Under a load of 25 kN, if the extension of the bar is 0.185 mm, what is the modulus of elasticity of the bar?

Sol: Given data



Let $L = 1000$

$L_1 = 1000 - 400$

$= 600$ mm

$L_2 = 400$ mm

$d_1 = 30$ mm

$d_2 = 10$ mm → bore diameter

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times 30^2}{4} = 225\pi = 706.86 \text{ mm}^2$$

$$A_2 = \frac{\pi (d_1^2 - d_2^2)}{4} = \frac{\pi (30^2 - 10^2)}{4} = 628.32 \text{ mm}^2$$

$$\delta L_1 = \frac{P L_1}{A_1 E_1} = \frac{P L_1}{A_1 E}$$

$$\delta L_2 = \frac{P L_2}{A_2 E}$$

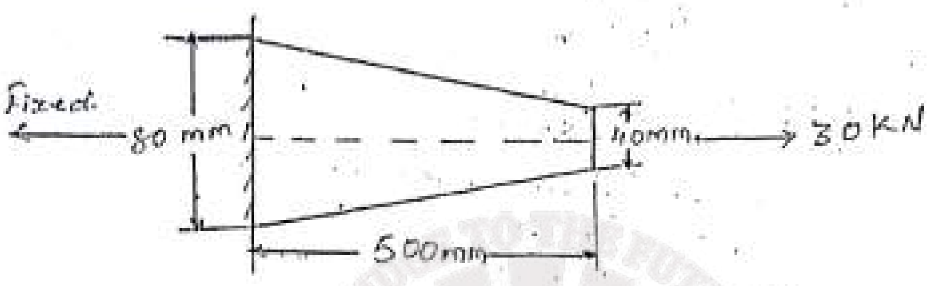
$$\delta L = \delta L_1 + \delta L_2 = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2}\right)$$

$$0.185 = \frac{25 \times 10^3}{E}\left(\frac{600}{706.86} + \frac{400}{628.32}\right)$$

$$E = 200.736 \text{ kN/mm}^2$$

source: diginotes.in

2) A steel bar of shown in fig having uniform thickness of 20mm is 200mm under an axial load of 30kN. Its extension is found to be 0.17mm. Find the 'E' in the problem.



Sol Given data. To find

$L = 500\ mm$ E (i) considering it as rectangular bar

$b_1 = 80\ mm$

$b_2 = 40\ mm$ (ii) E, considering as $\bigcirc^{lr}$ bar

$P = 30\ kN$

$\delta L = 0.17\ mm$

$t = 20\ mm$

(i) We know that

$$\delta L = \frac{PL}{t(b_1 - b_2)E} \log_e\left(\frac{b_1}{b_2}\right) \quad \text{for } \square^{lr} \text{ bar}$$

$$\Rightarrow E = \frac{PL}{t(b_1 - b_2)\delta L} \log_e\left(\frac{b_1}{b_2}\right)$$

$$= \frac{30 \times 10^3 \times 500}{20(80-40) \times 0.17} \log\left(\frac{80}{40}\right)$$

$$= 76.45 \times 10^3\ N/mm^2$$

(ii) $\delta L = \dfrac{4PL}{\pi d_1 d_2 E}$ for $\bigcirc^{lr}$ tapering bar

$$\Rightarrow E = \frac{4 \times 30 \times 10^3 \times 500}{\pi \times 80 \times 40 \times 0.17} = 35.107 \times 10^3\ N/mm^2$$

source: diginotes.in

Dec 2014/Jan 15

1.b) The tensile test was conducted on a mild steel bar. The following data was obtained from the test

Diameter of steel bar = 16 mm
Load @ proportionality limit = 72 KN
Gauge length of the bar = 80 mm
Load at failure = 80 KN
Diameter of the rod @ failure = 12 mm
Extension @ a load of 60 KN = 0.115 mm
Final gauge length of bar = 104 mm

Determine: (i) E (ii) Proportionality limit (ii) % elongation
(iii) True breaking stress

Sol:- Given data

$d = 16\ mm$
$P_1 = 72\ KN$ Load @ proportionality limit
$L = 80\ mm$
$P_f = 80\ KN$
$d_f = 12\ mm$ @ failure load
$\delta L' = 0.115\ mm$ @ 60 KN
$L' = 104\ mm$

(i) Youngs Modulus

$E = \frac{\sigma}{\varepsilon}$ within elastic limit

$\sigma = \frac{P}{A} = \frac{60 \times 10^3 \times 4}{\pi \times (16)^2} = \underline{298.41\ N/mm^2}$

$\varepsilon = \frac{\delta L}{L} = \frac{0.115}{80} = \underline{1.44 \times 10^{-3}}$

$E = \frac{\sigma}{\varepsilon} = \frac{298.41}{1.44 \times 10^{-3}} = \underline{207.59\ KN/mm^2}$

source: diginotes.in

ii) Proportionality Limit = $\sigma_{Limit} = \frac{\text{Load @ proportionality limit}}{\text{Original c/s Area}}$

$$= \frac{72 \times 10^3 \times 4}{\pi \times 16^2} = 358.1 \ N/mm^2$$

iii) True breaking stress = $\frac{\text{load @ failure}}{\text{Actual c/s Area @ failure}}$

$$= \frac{80 \times 10^3 \times 4}{\pi \times (12)^2} = 707.36 \ N/mm^2$$

iv) % elongation = $\frac{L'-L}{L} \times 100$

$$= \frac{104-80}{80} \times 100 = 30\ \%$$

Note

v) Nominal breaking stress = $\frac{\text{Load @ failure}}{\text{Original c/s Area}}$

$$= \frac{80 \times 10^3 \times 4}{\pi \times 16^2} = 397.89 \ N/mm^2$$

For the same problem, find yield stress & safe stress if load @ yield point = 75 KN & adopt FOS as 2.5

(vi) Yield stress = $\frac{\text{Load @ Yield point}}{\text{Area}}$

$$\sigma_y = \frac{75 \times 10^3 \times 4}{\pi \times 16^2} = 373.02 \ N/mm^2$$

(vii) Safe stress (or) working stress = σ_w = ?

W.K.T FOS = $\frac{\sigma_y}{\sigma_w} = \frac{\text{Yield stress}}{\text{Allowable stress}}$

$$\sigma_w = \frac{\sigma_y}{FOS} = \frac{373.02}{2} = 186.51 \ N/mm^2$$

$$\sigma_w = 186.51 \ N/mm^2$$

source: diginotes.in

Q) The extension of a bar uniformly tapering from a diameter of 'd+a' to 'd-a' in a length 'L' is calculated by treating it as a bar of uniform c/s of average diameter "d". What is the % error?

Sol: let P = load acting on the bar

L = length

d+a = larger dia

d-a = Smaller dia

E = Young's Modulus.

Actual extension under load P for a tapered circular bar is given as

$$\delta l = \frac{4PL}{\pi d_1 d_2 E} = \frac{4PL}{\pi (d+a)(d-a) E}$$

$$= \frac{4PL}{\pi (d^2-a^2) E}$$

If it is treated as a bar of uniform diameter 'd',

then extension $= \frac{PL}{\left(\frac{\pi d^2}{4}\right) E} = \frac{4PL}{\pi d^2 E}$

$$\therefore \% \text{ Error} = \frac{\frac{4PL}{\pi E(d^2-a^2)} - \frac{4PL}{\pi d^2 E}}{\left[\frac{4PL}{\pi E(d^2-a^2)}\right]} \times 100$$

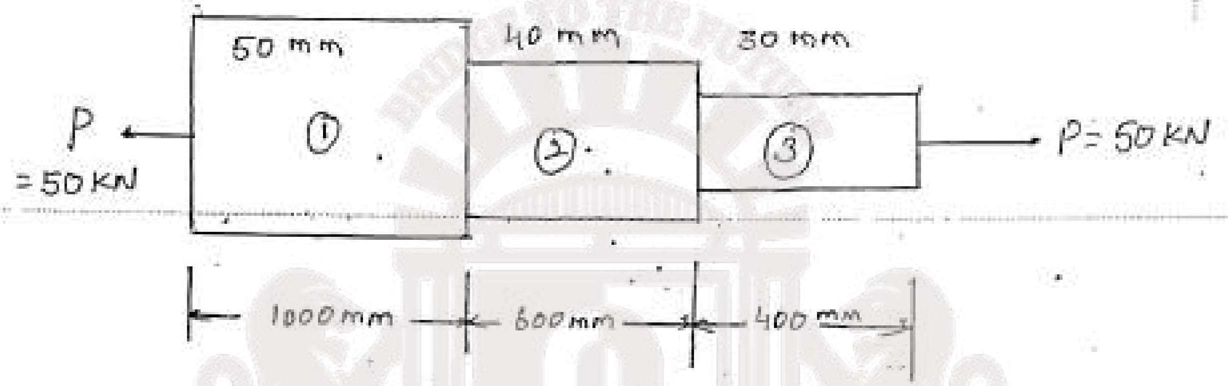
$$= \frac{\frac{1}{d^2-a^2} - \frac{1}{d^2}}{\left(\frac{1}{d^2-a^2}\right)} \times 100$$

$$= \left(1 - \frac{d^2-a^2}{d^2}\right) \times 100$$

source: diginotes.in

$\therefore$ % Error = $100 \frac{a^2}{d^2}$

Q) A stepped bar, circular cross section of 2mm length is subjected to an axial load of 50 kN. Find stress in each section, strain & deformation in each section & total deformation. Take E = 206 GPa.



Sol: Given data.

Section ①	Section ②	Section ③
$d_1 = 50$ mm	$d_2 = 40$ mm	$d_3 = 30$ mm
$l_1 = 1000$ mm	$l_2 = 600$ mm	$L_3 = 400$ mm

$E_1 = E_2 = E_3 = E = 206$ GPa

$= 206 \times 10^9$ N/m²

$= 206 \times 10^9$ N/10^6 mm²

$= 206 \times 10^9 \times 10^{-6}$ N/mm²

$\underline{E} = 206 \times 10^3$ N/mm²

Note: 1 GPa = 1 kN/mm² = 10^9 N/m²

Illy P = 50 kN.

$A_1 = \frac{\pi d_1^2}{4}$ $A_2 = \frac{\pi d_2^2}{4}$ $A_3 = \frac{\pi d_3^2}{4}$

$= 1963.5$ mm² $A_2 = 1256.64$ mm² $A_3 = 706.86$ mm²

source: diginotes.in

To find out

σ_1 σ_2 σ_3

ε_1 ε_2 ε_3 & $\delta L = \delta L_1 + \delta L_2 + \delta L_3$

δL_1 δL_2 δL_3

Section 1:

$$\sigma_1 = \frac{P_1}{A_1} = \frac{P}{A_1} = \frac{50 \times 10^3}{1963.5} = \underline{25.46} \text{ N/mm}^2$$

$$\varepsilon_1 = \frac{\delta L_1}{L_1} \text{ or } \frac{\sigma_1}{E_1}$$

$$= \frac{\sigma_1}{E} = \frac{25.46}{206 \times 10^3} = \underline{0.12 \times 10^{-3}}$$

$$\delta L_1 = L_1 \times \varepsilon_1 = 0.12 \times 10^{-3} \times 1000 = \underline{0.12} \text{ mm}$$

Section 2:

$$\sigma_2 = \frac{P}{A_2} = \frac{50 \times 10^3}{1256.64} = 39.79 \text{ N/mm}^2$$

$$\varepsilon_2 = \frac{\sigma_2}{E} = \frac{39.79}{206 \times 10^3} = \underline{0.19 \times 10^{-3}}$$

$$\delta L_2 = L_2 \times \varepsilon_2 = 600 \times 0.19 \times 10^{-3} = \underline{0.11} \text{ mm}$$

Section 3.

$$\sigma_3 = \frac{P}{A_3} = \frac{50 \times 10^3}{706.86} = 70.74 \text{ N/mm}^2$$

$$\varepsilon_3 = \frac{\sigma_3}{E} = \frac{70.74}{206 \times 10^3} = \underline{0.34 \times 10^{-3}}$$

$$\delta L_3 = L_3 \times \varepsilon_3 = 400 \times 0.34 \times 10^{-3} = 0.14 \text{ mm}$$

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3 = 0.12 + 0.11 + 0.14 = \underline{0.37 \text{ mm}}$$

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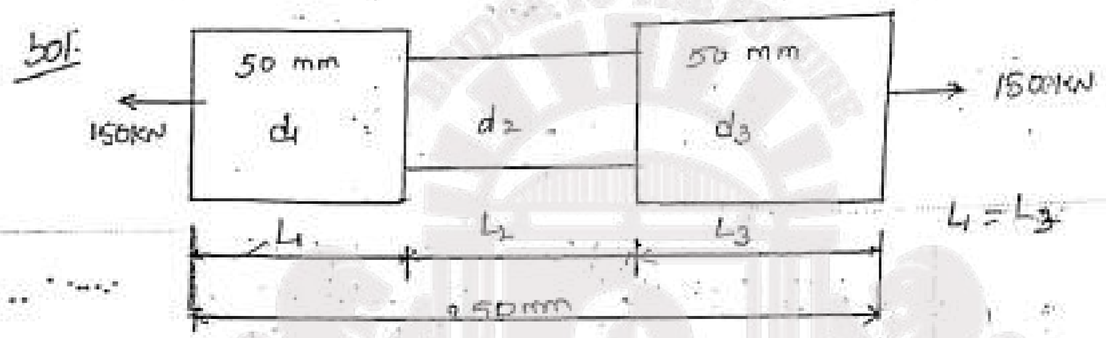
source: diginotes.in

8 The symmetrical steel bar of circular cross section as shown in fig is subjected to a tensile load of 150 KN. What must be the diameter of the middle section. If the stress limit is 200 MPa. What must be the length of the middle portion if the total elongation of the bar under the given load is 0.2mm, for the steel Young's Modulus is 206 GPa.

Sol:



Given data:

$P = 150$ KN

$d_1 = d_3 = 50$ mm

$L_1 = L_3$

$L = L_1 + L_2 + L_3 = 250$ mm

$= 2L_1 + L_2 = 250$

$L_2 = 250 - 2L_1$

$E = 206$ GPa $= 206 \times 10^3$ N/mm²

MPa $= 10^6$ N/m² $= 10^6$ N/10^{-6} mm²

MPa $=$ N/mm²

Stress in II section $= 200$ N/mm²

$$\sigma_2 = \frac{P}{A_2} = \frac{150 \times 10^3 \times 4}{\pi \times d_2^2}$$

$$200 = \frac{150 \times 10^3 \times 4}{\pi \times d_2^2}$$

$d_2 = 30.90$ mm $\Rightarrow$ $A_1 = 1963.5$ mm²

$A_2 = 749.91$ mm²

To find the length of the member for given extension in 0.2 mm

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3 = \frac{P L_1}{A_1 E_1} + \frac{P L_2}{A_2 E_2} + \frac{P L_3}{A_3 E_3}$$

here, $A_1 = A_3$, $E_1 = E_2 = E_3 = E$; $L_1 = L_3$

source: diginotes.in

$$\delta L = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right) = \frac{P}{E}\left(\frac{L_1 + L_3}{A_1} + \frac{L_2}{A_2}\right)$$

$$= \frac{P}{E}\left(\frac{2L_1}{A_1} + \frac{L_2}{A_2}\right) = \frac{P}{E}\left[\frac{250 - L_2}{A_1} + \frac{L_2}{A_2}\right]$$

$$= \frac{P}{E}\left[\frac{250 - L_2}{1963.5} + \frac{L_2}{A_2}\right]$$

$$0.2 = \frac{P}{E}\left[0.127 - \frac{L_2}{1963.5} + \frac{L_2}{A_2}\right]$$

$$\frac{0.2 \times E}{P} = 0.127 - \frac{L_2}{1963.5} + \frac{L_2}{A_2}$$

$$0.27 = 0.127 - \frac{L_2}{1963.5} + \frac{L_2}{749.91}$$

$$0.147 = L_2\left(\frac{1}{749.91} - \frac{1}{1963.5}\right)$$

$$L_2 = \frac{0.147}{0.0008}$$

$$L_2 = 179.2 \text{ mm}$$

$\therefore$ $2L_1 = 250 - 179.2$

$L_1 = 35.4$ mm

$L_3 = 35.4$ mm

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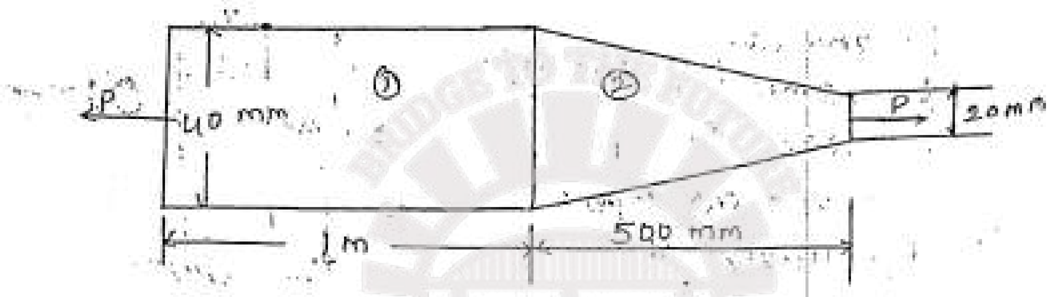
Q) A 1.5 m long steel bar is having uniform diameter of 40mm for a length of 1m in the next 0.5m its dia is gradually reducing to 20mm as shown in fig. Find the elongation bar if it is subjected to 160kN. Given E = 200 GPa.



P = 160kN

Sol: Given data.

Section 1

$P = 160\ kN$

$d_1 = 40\ mm$

$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times 40^2}{4}$

$=$

$L_1 = 1000\ mm$

$E = 200\ GPa = 200 \times 10^9\ N/m^2 = 200 \times 10^3\ N/mm^2$

Section 2

$P = 160\ kN$

$d_2 = 20mm$

$d_1 = 40mm.$

$A_2 = \frac{\pi (d_1 \times d_2)}{4}$

$L_2 = 500\ mm.$

To find out: $\delta L = \delta L_1 + \delta L_2 = ?$

$\delta L_1 = \frac{PL_1}{A_1 E} = \frac{160 \times 10^3 \times 1000 \times 4}{\pi \times 40^2 \times 200 \times 10^3}$

$= 0.637\ mm$

$\delta L_2 = \frac{PL_2}{A_2 E} = \frac{160 \times 10^3 \times 500 \times 4}{\pi \times 40 \times 20 \times 2 \times 10^5}$

$= 0.637\ mm$

$\therefore \delta L = \delta L_1 + \delta L_2 = 0.637 + 0.637$

$\delta L = 1.274\ mm$

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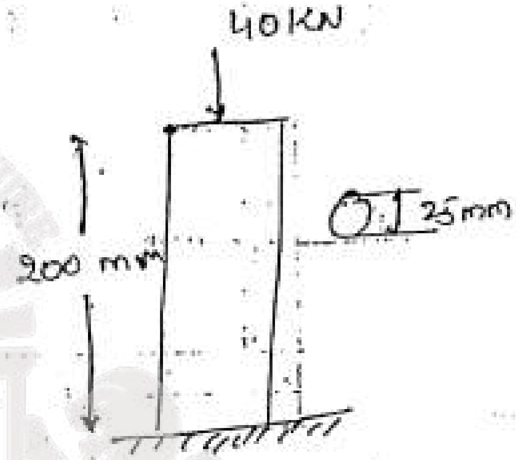
source: diginotes.in

Q) A 200 mm long circular bar has circular cross-section of 25 mm diameter. Determine the stress, deformation & axial strain induced in the bar, when it is subjected to a compressive force 40 kN. Take $E = 200$ GPa.



Sol: Given data

$L = 200$ mm

$d = 25$ mm

$E = 200$ GPa

$= 200 \times 10^3 \; N/mm^2$

$P = 40 \times 10^3$ N

$\therefore A = \frac{\pi d^2}{4} = \frac{\pi \times 25^2}{4} = 491 \; mm^2$

To find out: σ, ε, δL

i) $\sigma = \frac{P}{A} = \frac{40 \times 10^3}{491} = 81.5 \; N/mm^2$

ii) $\varepsilon = \frac{\sigma}{E} = \frac{81.5}{2 \times 10^5} = 4.075 \times 10^{-4}$

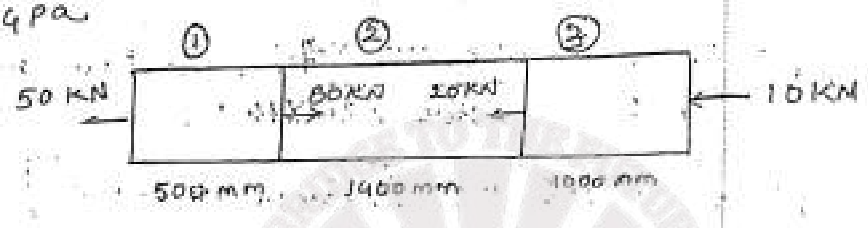
iii) $\delta L = L \times \varepsilon = 200 \times 4.075 \times 10^{-4}$

$= 0.082$ mm

source: diginotes.in

Principle of Superposition

(1) A brass bar having cross-sectional area $300\,mm^2$ is subjected to axial forces as shown in fig. find the total elongation of the bar. Take E as 84 GPa



Sol: Given data.

①	②	③
$L_1 = 500\,mm$	$L_2 = 1400\,mm$	$L_3 = 1000\,mm$

$d_1 = d_2 = d_3 = d$

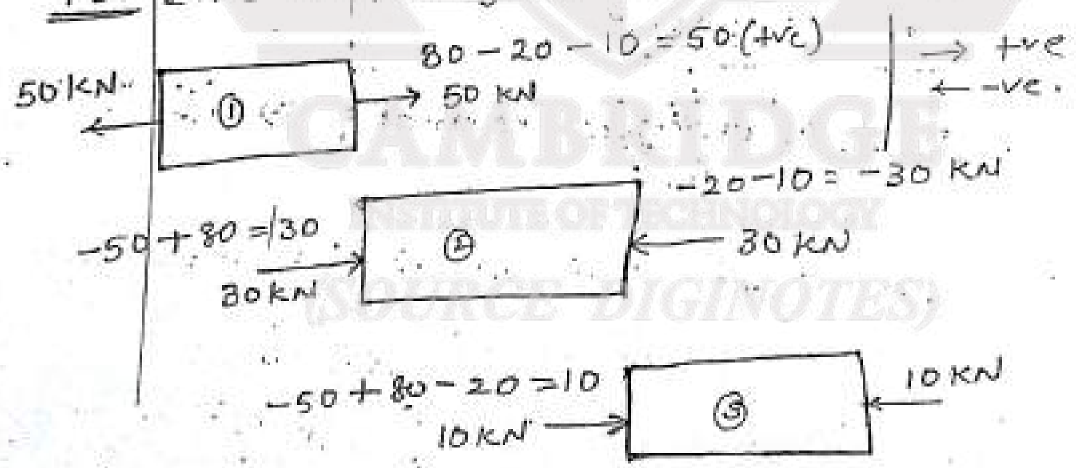
$A_1 = A_2 = A_3 = 300\,mm^2$

$E_1 = E_2 = E_3 = E = 84\,GPa$

$= 84 \times 10^9\ N/m^2$

$= 84 \times 10^3\ N/mm^2$

FBD [free body diagram for each section]



Note Section ① under Tension (Tensile load)

Section ② & ③ under compression (compressive load)

Take δL = +ve under tension [↑ in length]

δL = −ve under compression [↓ in length]

source: diginotes.in

$$\delta L_1 = \frac{P_1 L_1}{A_1 E} = \frac{50 \times 10^3 \times 500}{300 \times 84 \times 10^3} = \underline{0.99} \text{ mm}$$

Section (2)

$$\delta L_2 = \frac{P_2 L_2}{A E} = \frac{30 \times 10^3 \times 1400}{300 \times 84 \times 10^3} = 1.67 \text{ mm}$$

$$\delta L_2 = -1.67 \text{ mm} \quad [\because \text{compressive load}]$$

Section (3)

$$\delta L_3 = \frac{P_3 L_3}{A E} = \frac{10 \times 10^3 \times 1000}{300 \times 84 \times 10^3} = 0.397 \text{ mm}$$

$$\therefore \delta L_3 = -0.397 \text{ mm} \quad [\because \text{Compressive}]$$

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$

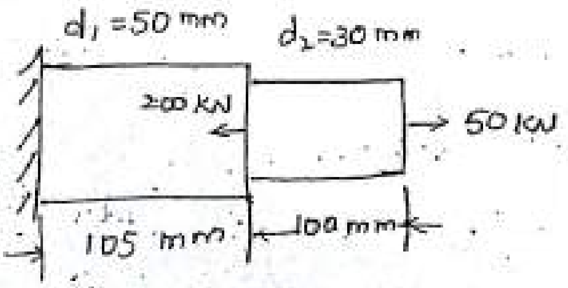
$$= 0.99 - 1.67 - 0.397$$

$$\underline{\delta L = -1.08 \text{ mm}}$$

$\therefore$ Body undergone deformation of $\underline{1.08}$ m.

Q) Find the reaction, individual stress & deformation in the structure.

E = 206 GPa.



Sol: <u>Given data</u>

$E = 206$ GPa

$= 206 \times 10^3$ N/mm^2

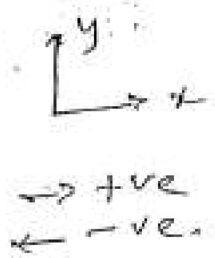
$L_1 = 105$ mm

$L_2 = 100$ mm

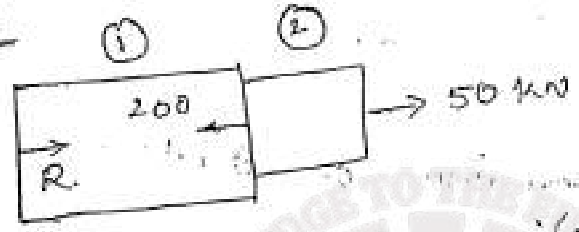
source: diginotes.in

$d_1 = 50$ mm

$d_2 = 30$ mm



FBD



For a body to be under equilibrium condition

$\sum F_x = 0$ [Forces in x-direction = F_x]

$+R - 200 + 50 = 0$

$R - 150 = 0$ [Reaction]

$R = 150$ kN

Individual stress

$\sigma_1 = \frac{P_1}{A_1} = \frac{150 \times 10^3 \times 4}{\pi \times 50^2} = 76.394 \ N/mm^2$

$\sigma_2 = \frac{P_2}{A_2} = \frac{50 \times 10^3 \times 4}{\pi \times 30^2} = 70.736 \ N/mm^2$

Individual deformation

$\delta_1 = \frac{P_1 L_1}{A_1 E} = \frac{\sigma_1 L_1}{E} = \frac{76.394 \times 105}{206 \times 10^3}$

$= 0.03894$ mm

source: diginotes.in

$$\delta l_2 = \frac{P_2 L_2}{A_2 E} = \frac{\sigma_2 L_2}{E} = \frac{100 \times 70.736}{206 \times 10^3}$$

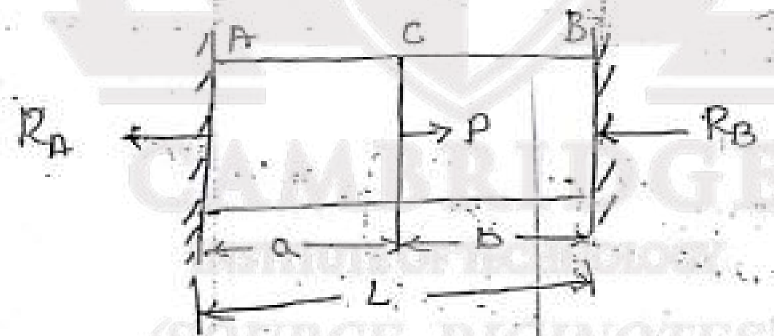
$\delta l_2 = \underline{0.03434}$ mm.

∴ Overall deformation $\delta L = \delta l_1 + \delta l_2$

$= \underline{0.0733}$ mm.

④ A homogeneous rod of constant c/s is attached to unyielding supports. It carries an axial load 'P' applied as shown in fig. Determine the reaction @ A & B.

Sol:



Applying the condition of static equilibrium.

$-R_A - R_B + P = 0$

$R_A + R_B = P \longrightarrow ①$

* When the bar is fixed rigidly between supports, extension in the section AC = contraction in the section CB

FBD $\qquad P - R_B = R_A$ (from ①)

R_A ... R_A

$P \rightarrow R_A$ & R_B

$$\delta_{AC} = \frac{R_A \cdot a}{A \cdot E} \quad ; \quad \delta_{CB} = \frac{R_B \times b}{A \times E}$$

Given $\delta_{AC} = \delta_{CB}$

$$\frac{R_A\, a}{A E} = \frac{R_B\, b}{A E}$$

$$\frac{R_A}{R_B} = \frac{b}{a}$$

$$\therefore R_A = \left(\frac{b}{a}\right) R_B \rightarrow (2)$$

Substitute in (1)

$$\left(\frac{b}{a}\right) R_B + R_B = P$$

$$\Rightarrow R_B\left(\frac{b}{a} + 1\right) = P$$

$$\Rightarrow R_B\left(\frac{b+a}{a}\right) = P$$

$$\Rightarrow \boxed{R_B = \frac{P \cdot a}{b+a}}$$

$$\therefore R_A = \left(\frac{b}{a}\right)\left(\frac{P \cdot a}{b+a}\right) = \frac{P b}{a+b}$$

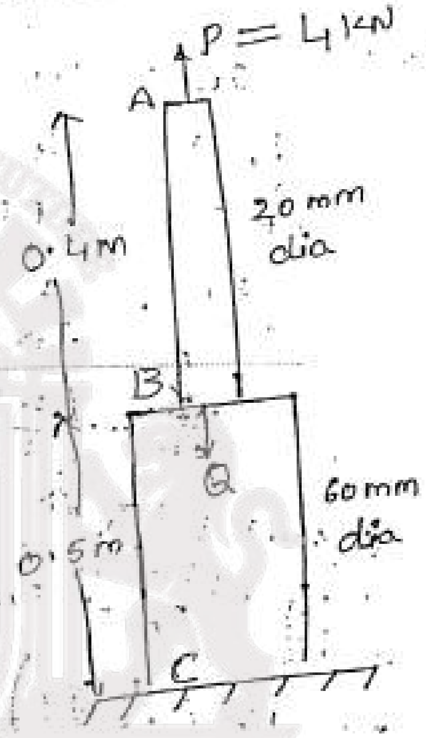
$$\boxed{R_A = \frac{P \cdot b}{a+b}}$$

source: diginotes.in

Q) In the given fig, AB & BC are made of aluminium for which E = 70 GPa. Knowing that the magnitude of P is 4 kN. Determine

a) The value of Q so that the deflection @ A = 0

b) The corresponding deflection @ B.



Sol: Given data

Portion AB

$L_{AB} = 0.4$ m

$d_{AB} = 20$ mm

Portion BC

$L_{BC} = 0.5$ m

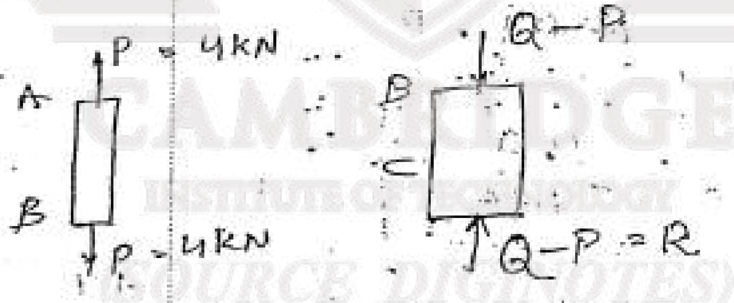
$d_{BC} = 60$ mm

(a) deflection @ A = 0

P = 4 kN

$\Rightarrow \delta L = 0$

FBD



$\delta L = \delta L_1 + (-\delta L_2) \Rightarrow \delta L = \delta L_1 - \delta L_2$

∵ BC portion under compression

$\Rightarrow 0 = \frac{P L_{AB}}{A_{AB} E} - \frac{(Q-P) L_{BC}}{A_{BC} E}$

$\Rightarrow \frac{P \times L_{AB}}{A_{AB} E} = \frac{(Q-P) L_{BC}}{A_{BC} E}$

source: diginotes.in

$\Rightarrow \frac{4\times10^3\times 0.4\times10^3\times4}{\pi\times20^2\times70\times10^3} = \frac{(Q-4\times10^3)\times 0.5\times10^3\times4}{\pi\times60^2\times70\times10^3}$

$\Rightarrow 28800.00 = Q-40\times10^3$

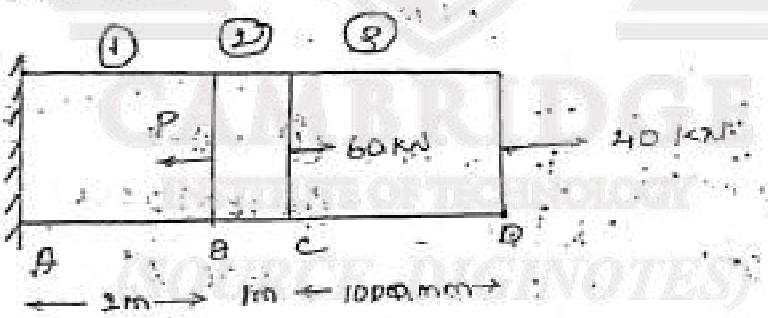
$\Rightarrow$ $\boxed{Q = 32.8\ kN}$

b) $\delta_{BC} = \frac{(Q-P)L_{BC}}{A_{BC}\times E}$

$= \frac{(32.8-4)10^3\times500\times4}{\pi\times60^2\times70\times10^3}$

$\delta_{BC} = 0.0727$ mm

Q) Determine the magnitude of the load P necessary to produce '0' net change in the length of the straight bar shown in Fig. Area of c/s = 400 mm².



Sol: Given data.

c/s Area of bar = A = 400 mm²

Length of section 1 = L_1 = 2m = 2000 mm

Length " " 2 = L_2 = 1m = 1000 mm

" " " 3 = L_3 = 1 m = 1000 mm.

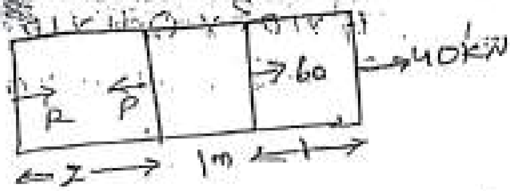
* Let the reaction @ the suppor = R & acts towards right.

for equilibrium of the bar, $\Sigma F_x = 0$

change in length = $\delta L = 0$ (Given)

source: diginotes.in

from fig



$R - P + 60 + 40 = 0$

$R - P + 100 = 0$

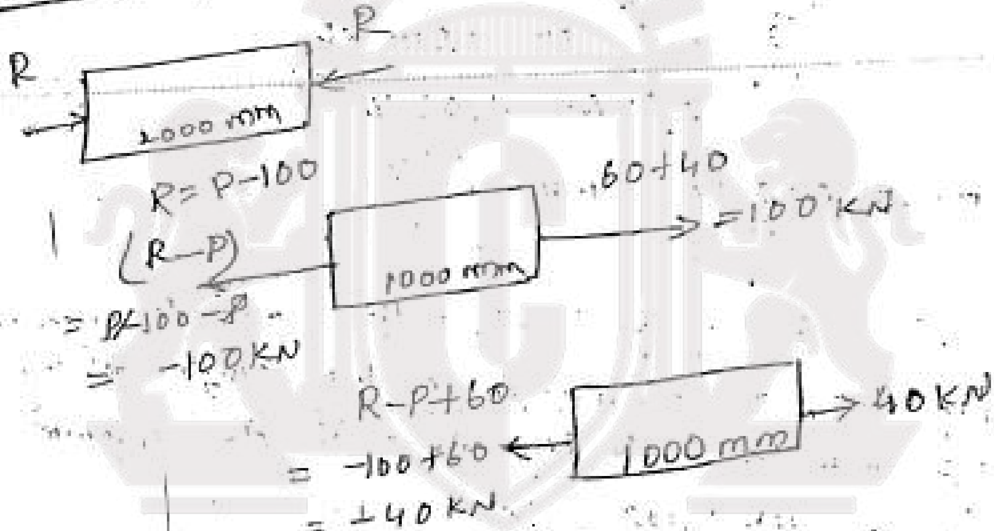
$R - P = -100$

$R = P - 100$

$\rightarrow$ +ve

$\leftarrow$ −ve.

FBD



∴ Deformation in section ① $= \delta L_1$

$\Rightarrow \delta L_1 = \frac{R L_1}{A_1 E_1} = \frac{(P-100)\,2000}{400 \times E}$

IIly $\delta L_2 = \frac{100 \times L_2}{AE} = \frac{100 \times 1000}{AE}$

IIly $\delta L_3 = \frac{40 L_3}{AE} = \frac{40 \times 1000}{AE}$

But $\delta L = \delta L_1 + \delta L_2 + \delta L_3$ $\quad (\because \delta L = 0)$

$0 = \delta L_1 + \delta L_2 + \delta L_2$

source: diginotes.in

$$\Rightarrow 0 = \frac{(P-100)\,2000}{AE} + \frac{100\times10^3}{AE} + \frac{40\times10^3}{AE}$$

$$\Rightarrow 0 = 2000P - 2\times10^3\times10^2 + 10^5 + 40\times10^3$$

$$\Rightarrow P = 170\ KN$$

∴ Load necessary to produce zero net change in the length of the bar = $P = 170\ KN$

8) A member ABCD is subjected to point loads P_1, P_2, P_3 & P_4 as shown in fig. Calculate the force P_2 necessary for equilibrium, if $P_1 = 45\ KN$, $P_3 = 450\ KN$ & $P_4 = 130\ KN$. Determine the total elongation of the member, assuming the modulus of elasticity to be $2.1\times10^5\ N/mm^2$

Sol: Given data

$P_1 = 45\ KN$ $\quad P_2 = ?$

$P_3 = 450\ KN$ $\quad \delta L = ?$

$P_4 = 130\ KN$

$E = 2.1\times10^5\ N/mm^2$

$L_{AB} = 1200\ mm$

$L_{BC} = 600\ mm$

$L_{CD} = 900\ mm$

$A_1 = 625\ mm^2$ = area of AB

$A_2 = 2500\ mm^2$ = area of BC

$A_3 = 1250\ mm^2$ = area of CD

source: diginotes.in

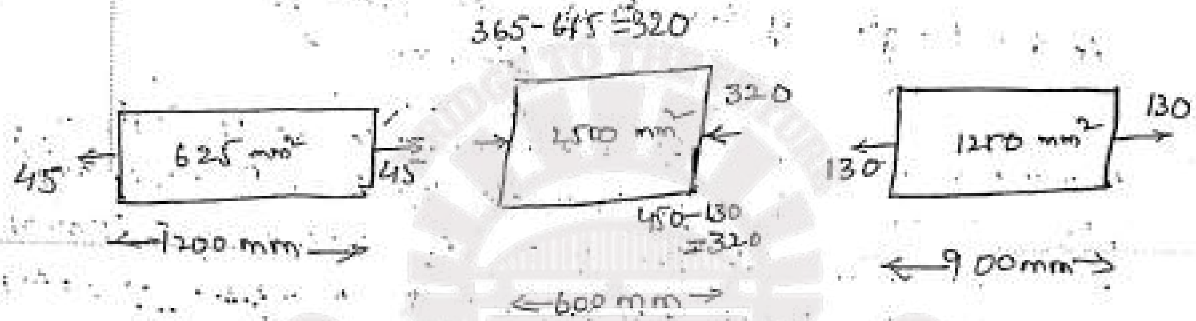
Under equilibrium of bar.

$$\Sigma F_x = 0 = -P_1 + P_2 - P_3 + P_4$$

$$0 = -45 + P_2 - 450 + 130$$

$$P_2 = +365 \text{ KN}$$

$$P_2 = 365 \text{ KN} (\rightarrow)$$



To find δL:

$\delta L = \delta L_1 + \delta L_2 + \delta L_3$ if all sections under tensile.

$$\delta L_1 = \frac{P_1 L_1}{A_1 E} = \frac{45 \times 10^3 \times 1200}{625 \times 2.1 \times 10^5} = 411.43 \times 10^{-3}$$

$$\delta L_2 = \frac{P_2 L_2}{A_2 E} = \frac{320 \times 600 \times 10^3}{2500 \times 2.1 \times 10^5} = 365.71 \times 10^{-3}$$

$$\delta L_3 = \frac{P_3 L_3}{A_3 E} = \frac{130 \times 10^3 \times 900}{1250 \times 2.1 \times 10^5} = 445.71 \times 10^{-3}$$

$\delta L = \delta L_1 - \delta L_2 + \delta L_3$ ($\because \delta L_2$ under compression)

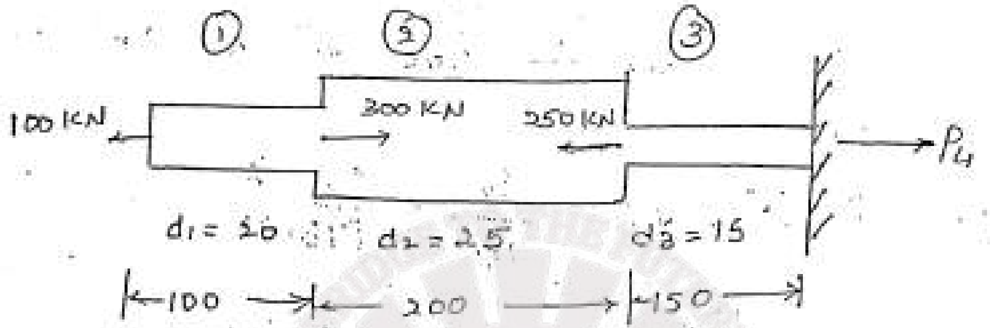
$\delta L = 0.49143$ mm (elongation)

Q) Determine the stresses in various segments of the circular bar as shown in fig.

Q) Compute its total elongation assuming E of steel to be 195 GPa.

source: diginotes.in

(b) Determine the length of the middle segment so that the bar length does not change under the applied load.



All dimensions are in mm

Sol:- Given data.

$E = 195\ GPa = 195 \times 10^9\ N/m^2$

$= 195 \times 10^3\ N/mm^2$

① $d_1 = 20\ mm$, $L_1 = 100\ mm$

$A_1 = \frac{\pi d_1^2}{4}$

$= 314.16\ mm^2$

② $d_2 = 25\ mm$, $L_2 = 200\ mm$

$A_2 = \frac{\pi d_2^2}{4}$

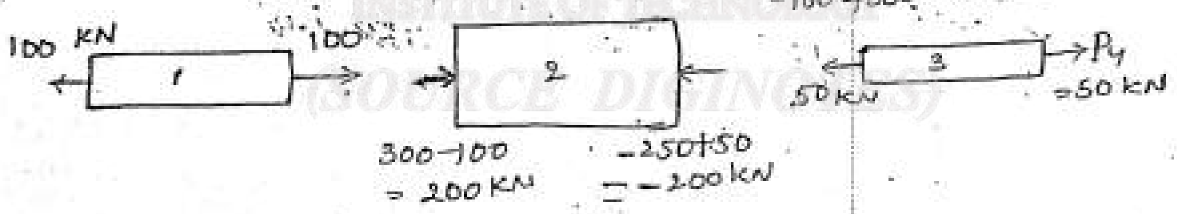
$A_2 = 490.87\ mm^2$

③ $d_3 = 15\ mm$, $L_3 = 150\ mm$

$A_3 = \frac{\pi d_3^2}{4}$

$A_3 = 176.71\ mm^2$

$-100 + 300 - 250 = 50\ kN$



Under equilibrium

$-100 + 300 - 250 + P_4 = 0$

$P_4 = 50\ KN$

Stress: Stress in section ①, $\sigma_1 = \frac{P_1}{A_1} = \frac{100 \times 10^3}{314.16}$

$\sigma_1 = 318.31\ N/mm^2$

source: diginotes.in

Stress in ②, $\sigma_2 = \frac{P_2}{A_2} = \frac{200\times10^3}{490.87} = 407.44\ N/mm^2$

Note: σ_2 is a compressive stress due to compressive load

Stress in ③, $\sigma_3 = \frac{P_3}{A_3} = \frac{50\times10^3}{176.71} = 282.94\ N/mm^2$

a) Total elongation :

$\delta L = \delta L_1 + \delta L_2 + \delta L_3$

$\delta L_1 = \frac{P_1 L_1}{A_1 E} = \frac{\sigma_1 L_1}{E} = \frac{318.31\times100}{195\times10^3}$

$\delta L_1 = 0.163\ mm$

$\delta L_2 = \frac{P_2 L_2}{A_2 E} = \frac{\sigma_2 L_2}{E} = \frac{407.44\times200}{195\times10^3}$

$\delta L_2 = 0.418\ mm$

$\delta L_3 = \frac{P_3 L_3}{A_3 E} = \frac{\sigma_3 L_3}{E} = \frac{282.94\times150}{195\times10^3}$

$\delta L_3 = 0.218\ mm$

$\delta L = \delta L_1 + \delta L_2 + \delta L_3$

$= 0.163 - 0.418 + 0.218$ $(\because \delta L_2 = -ve)$

$\delta L = -0.037\ mm$

$\therefore \delta L = 0.037\ mm$ (contraction)

source: diginotes.in

(b) $L_2 = ?$ when. $\delta L = 0$

$\Rightarrow \delta L = \delta L_1 + \delta L_2 + \delta L_3$

$$0 = \frac{\sigma_1 L_1}{E} - \frac{\sigma_2 L_2}{E} + \frac{\sigma_3 L_3}{E}$$

$$\Rightarrow 0 = 0.163 - \frac{\sigma_2 \cdot L_2}{E} + 0.218$$

$$\Rightarrow 0 = 0.381 - \frac{407.44 \times L_2}{195 \times 10^3}$$

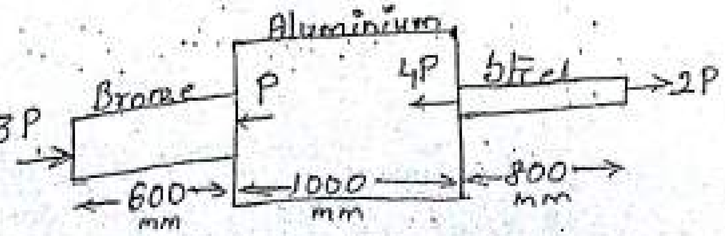
$\Rightarrow L_2 = 182.35$ mm

$\therefore$ The length of middle bar segment under zero deformation is $L_2 = 182.35$ mm

Q) A compound bar consisting of Bronze, Al & steel segments is loaded axially as shown in fig. Determine the maximum allowable value of 'P', if the change in length of bar is not to be exceeded 2mm & the working stresses in each material of the bar indicated in table below is not to be exceeded. [15M]

Material	Area (mm^2) (A)	E MPa $\times 10^5$	Working stress σ_w (MPa)
Bronze	450	0.83	120
Aluminium	600	0.70	80
steel	300	2	140

Sol: Given data.

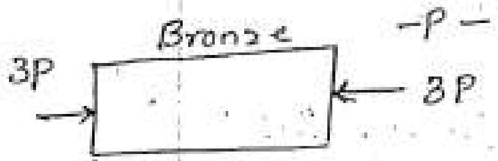


source: diginotes.in

Sol:

FBD, with Given data

$-P-4P+2P=-3P$



$L_B = 600$ mm

$A_B = 450$ mm^2

$E_B = 0.83\times10^5$ MPa $= 0.83\times10^5$ N/mm^2

$\sigma_{wB} = 120$ MPa $= 120$ N/mm^2

$3P-P=2P$ $\qquad -4P+2P=-2P$

$L_{Al} = 1000$ mm

$A_{Al} = 600$ mm^2

$E_{Al} = 0.70\times10^5$ N/mm^2

$\sigma_{w(Al)} = 80$ MPa $= 80$ N/mm^2

$= 80$ N/mm^2

$3P-P-4P$

$L_S = 800$ mm

$A_S = 300$ mm^2

$E_S = 2\times10^5$ N/mm^2

$\sigma_{wS} = 140$ N/mm^2

$\delta L = 2$ mm

$\delta L = \delta L_1 + \delta L_2 + \delta L_3 = \delta L_B + \delta L_{Al} + \delta L_S$

$$\delta L_B = \frac{P_B L_B}{A_B E_B}$$

$$= \frac{3P \times 600}{450 \times 0.83\times10^5} = 4.819\times10^{-5}P$$

source: diginotes.in

$\delta L_{Al} = \left(\frac{PL}{AE}\right)_{Al}$

$= \frac{2P \times 1000 \times 10^{-5}}{600 \times 0.70} = 4.762 \times 10^{-5} P$

$\delta L_s = \left(\frac{PL}{AE}\right)_s$

$= \frac{2P \times 800}{300 \times 2 \times 10^5} = 2.667 \times 10^{-5} P$

$\delta L = P(-4.819 - 4.762 + 2.667) \times 10^{-5}$

$\delta L = -6.914 \times 10^{-5} P$

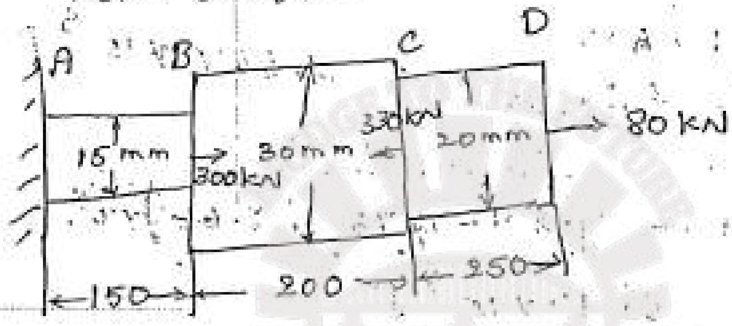
$2 = -6.914 \times 10^{-5} P$

$\Rightarrow P = 28.93 \text{ KN}$

CAMBRIDGE INSTITUTE OF TECHNOLOGY (SOURCE: DIGINOTES)

source: diginotes.in

Q) A mild steel circular bar has 3 segments as shown in fig. find
i) The total elongation of the bar
ii) The length of the middle segment to have zero elongation of the bar.
iii) The diameter of the last segment to have zero elongation of the bar. Take $E = 205\,GPa$



Sol: Given data:

$L_{AB} = 150$ mm $L_{BC} = 200$ $L_{CD} = 250$ mm

$d_{AB} = 15$ mm $d_{BC} = 30$ mm $d_{CD} = 20$ mm

$E = 205 \times 10^9\ N/m^2$

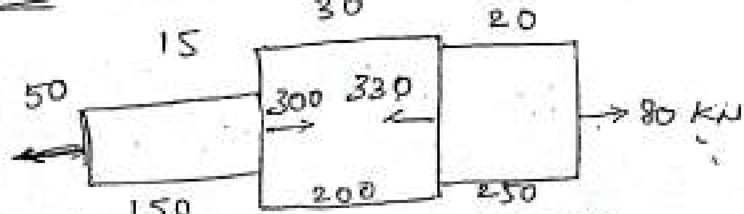
$= 205 \times 10^3\ N/mm^2$

For a body to be in equilibrium, $\Sigma F_x = 0$

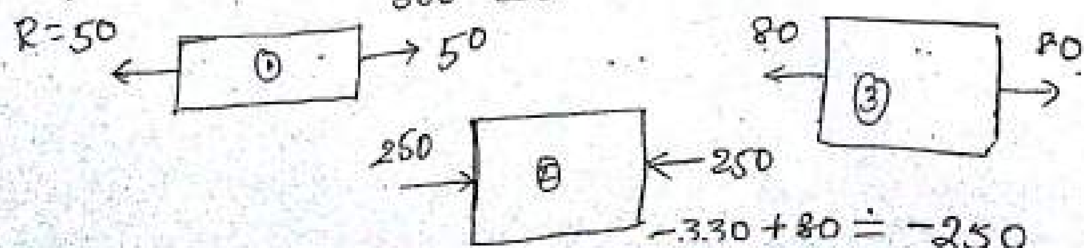
$\Rightarrow \Sigma F_x = 0 = +R + 300 - 330 + 80$

$\Rightarrow R = -50$ kN

FBD



$300 - 330 + 80 = 50$



$-330 + 80 = -250$

source: diginotes.in

(i) $\delta L = ?$

$\delta L = \delta L_{AB} + \delta L_{BC} + \delta L_{CD}$

$\delta L_{AB} = \left(\frac{PL}{AE}\right)_{AB}$

$= \frac{50 \times 150 \times 4 \times 10^3}{\pi \times 15^2 \times 205 \times 10^3} = \underline{0.207 \text{ mm}}$

$\delta L_{BC} = \left(\frac{PL}{AE}\right)_{BC} = \frac{250 \times 10^3 \times 4 \times 200}{\pi \times 30^2 \times 205 \times 10^3}$

$\therefore \delta L_{BC} = \underline{0.345 \text{ mm}}$

$\delta L_{CD} = \left(\frac{PL}{AE}\right)_{CD} = \frac{80 \times 10^3 \times 4 \times 250}{\pi \times 20^2 \times 205 \times 10^3}$

$\therefore \delta L_{CD} = \underline{0.311 \text{ mm}}$

$\delta L = \delta L_{AB} - \delta L_{BC} + \delta L_{CD}$ [$\because \delta L_{BC}$ under compression]

$= 0.207 - 0.345 + 0.311 = \underline{0.173 \text{ mm}}$

ii) Length of middle segment = ? if $\delta L = 0$

$0 = \delta L_{AB} + \delta L_{BC} + \delta L_{CD}$

$0 = 0.207 + \delta L_{BC} + 0.311 \Rightarrow \underline{\delta L_{BC} = -0.518 \text{ mm}}$

$\Rightarrow \delta L_{BC} = \left(\frac{PL}{AE}\right)_{BC} = \frac{250 \times 10^3 \times 4 \times L}{\pi \times 30^2 \times 205 \times 10^3}$

$\Rightarrow +0.518 = 0.0017 L$

$\Rightarrow \boxed{L = 304.706 \text{ mm}}$

iii) Dia of segment (3), $\delta L = 0$

$\Rightarrow 0 = \delta L_{AB} - \delta L_{BC} + \delta L_{CD}$

$\Rightarrow 0 = 0.207 - 0.345 + \delta L_{CD}$

$\Rightarrow \delta L_{CD} = 0.138 \text{ mm}$

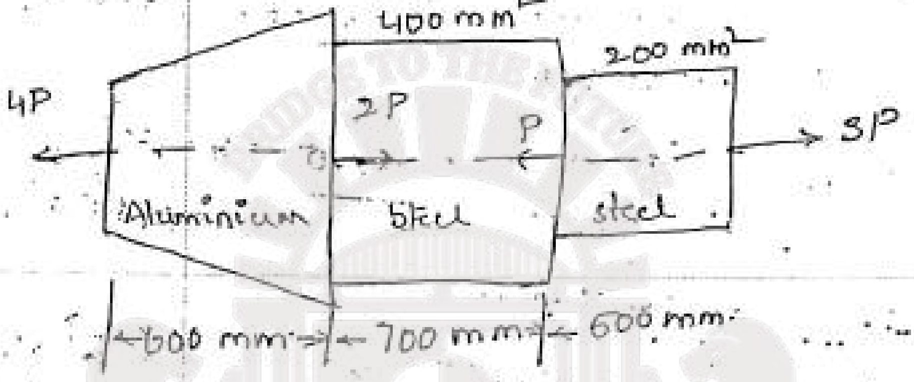
$\Rightarrow 0.138 = \frac{80 \times 10^3 \times 4 \times 250}{\pi \times d^2 \times 205 \times 10^3}$

$\Rightarrow d^2 = 900.134$

$d = \underline{30.002 \text{ mm}}$

source: diginotes.in

Q) A round bar with stepped portion is subjected to the forces as shown. Determine the magnitude of force P such that net deformation in the bar does not exceed 1mm. E for steel is 200 GPa and that for aluminium is 70 GPa. Big end diameter & small end diameter of the tapering bar are 40 mm & 12.5 respectively.



Sol: Given data:

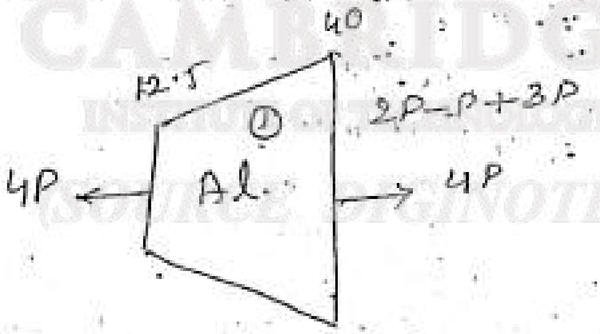
$E_s = 200\ GPa = 200 \times 10^3\ N/mm^2$

$E_{al} = 70\ GPa = 70 \times 10^3\ N/mm^2$

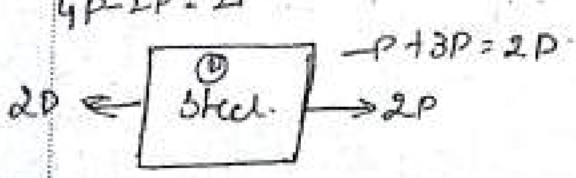
For equilibrium

$\Sigma F_x = 0 \Rightarrow -4P + 2P - P + 3P = 0$

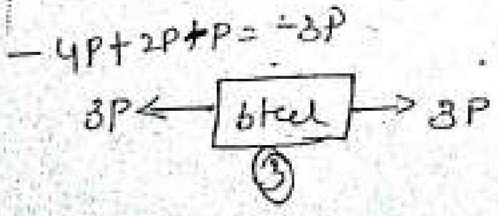
∴ FBD



$L_1 = 600\ mm$
$d_1 = 12.5\ mm$
$d_2 = 40\ mm$
$E_1 = 70 \times 10^3\ N/mm^2$



$L_2 = 700\ mm$
$A_2 = 400\ mm^2$
$E_2 = 2 \times 10^5\ N/mm^2$



$L_3 = 500\ mm$
$A_3 = 200\ mm^2$
$E_3 = 2 \times 10^5\ N/mm^2$

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$\delta L = \delta L_1 + \delta L_2 + \delta L_3$

$\Rightarrow \delta L_1 = \left(\frac{PL}{AE}\right)_1 = \frac{4P_1L_1}{\pi d_1 d_2 E} = \frac{4 \times 4P \times 600}{\pi \times 7 \times 10^4 \times 40 \times 125}$

$\delta L_1 = 8.73 \times 10^{-8} \cdot P$

$\Rightarrow \delta L_2 = \left(\frac{PL}{AE}\right)_2 = \frac{P_2 L_2}{A_2 E_2} = \frac{2 \times P \times 700}{400 \times 2 \times 10^5}$

$\delta L_2 = 1.75 \times 10^{-5} P$

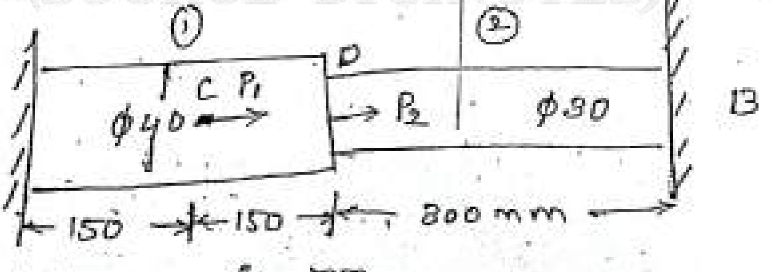
$\Rightarrow \delta L_3 = \left(\frac{PL}{AE}\right)_3 = \frac{5P \times 500}{200 \times 2 \times 10^5} = 3.75 \times 10^{-5} P$

$\therefore \delta L = \delta L_1 + \delta L_2 + \delta L_3$

$1 = (8.73 + 1.75 + 3.75) \times 10^{-5} P$

$\Rightarrow$ **$P = 7027.4$ N**

Q) A stepped bar of steel, held b/w 2 supports as shown in fig. is subjected to loads $P_1 = 80$ kN & $P_2 = 60$ kN. Find the reactions developed @ the ends A & B.



Sol: All dimensions are in mm.

Given data

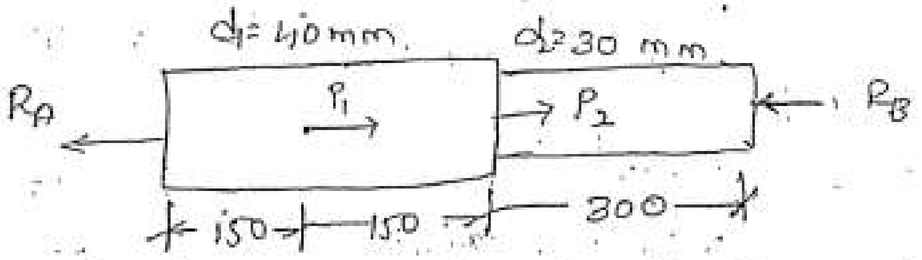
$L_{AC} = 150$ mm $L_{CD} = 150$ $L_{DB} = 300$ mm

$d_1 = 40$ mm $d_2 = 30$ mm

$P_1 = 80$ kN $P_2 = 60$ kN

source: diginotes.in

End reactions



Under equilibrium condition, $\Sigma F_x = 0$

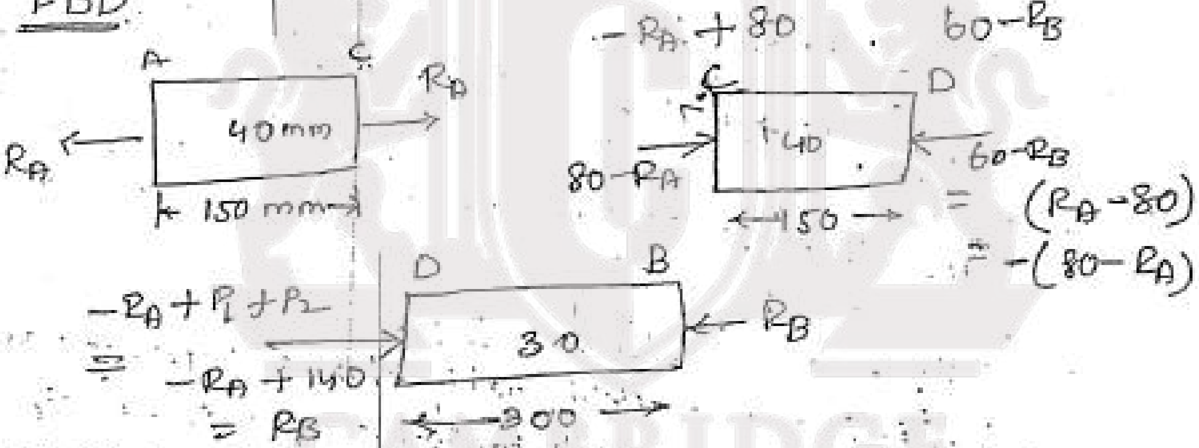
$$P_1 + P_2 = R_A + R_B$$

$$80 + 60 = R_A + R_B$$

$$\therefore R_A + R_B = 140 \longrightarrow (1)$$

$$R_A = 140 - R_B$$

FBD



Since supports are fixed on the both sides.

$$\delta l = 0$$

$$0 = \delta l_{AC} + \delta l_{CD} + \delta l_{DB}$$

$$0 = \left(\frac{FL}{AE}\right)_{AC} + \left(-\frac{FL}{AE}\right)_{CD} + \left(-\frac{FL}{AE}\right)_{DB}$$

$$0 = \frac{R_A \times 150 \times 4}{\pi \times 40^2 \times E} - \frac{(80 - R_A) \times 150 \times 4}{\pi \times 40^2 \times E} - \frac{R_B \times 4 \times 300}{\pi \times 30^2 \times E}$$

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$$\Rightarrow \quad 0 = \frac{150\times4}{40^2}R_A - \frac{150\times4}{40^2}\times80 + \frac{150\times4}{40^2}R_A - \frac{300\times4}{30^2}R_B$$

$$\Rightarrow \quad 0 = 2\times\frac{150\times4}{40^2}R_A - \frac{150\times4\times80}{40^2} - \frac{1200R_B}{30^2}$$

$$\Rightarrow \quad 0 = (R_A\times0.75) - (30) - (R_B\times1.333)$$

But $R_A + R_B = 140$ & substitute in above equation

$$\boxed{R_A = 104\ kN}$$

$$R_B = 140 - 104$$

$$\boxed{R_B = 36\ kN}$$

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Uniform bar subjected to loads along length

A uniform cross section bar of 20 mm diameter is subjected to loads as shown. Find the total elongation of the bar & the maximum stress in the bar. $E = 200\,GPa$ (All lengths are in mm).

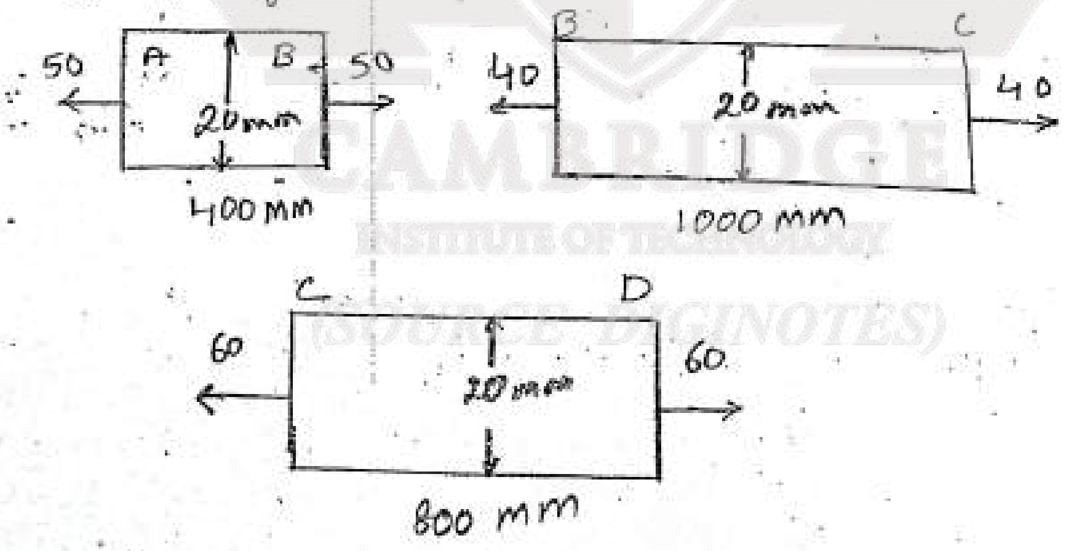
Sol:- Uniform c/s area of bar $\Rightarrow A = \frac{\pi D^2}{4}$

dia of bar = 20 mm [Given]

$E = 200\,GPa$

FBD

(i) FBD of section AB



w.k.t, $\delta L = \delta L_1 + \delta L_2 + \delta L_3$

where $\delta L_1 = \frac{P_1 L_1}{A_1 E_1}$, $\delta L_2 = \frac{P_2 L_2}{A_2 E_2}$, $\delta L_3 = \frac{P_3 L_3}{A_3 E_3}$

But $A_1 = A_2 = A_3$ & $E_1 = E_2 = E_3$

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$$\therefore \delta L = \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} + \frac{P_3 L_3}{AE}$$

$$= \frac{1}{AE}\left(P_1 L_1 + P_2 L_2 + P_3 L_3\right)$$

$$= \frac{4 \times 10^{-9} \times 10^{3}}{\pi (20 \times 10^{-3})^2 \times 200}\left[50 \times 400 + 40 \times 1000 + 60 \times 800\right] \times 10^{-3}$$

$$\delta L = 1.719 \text{ mm}$$

Maximum Stress = ?

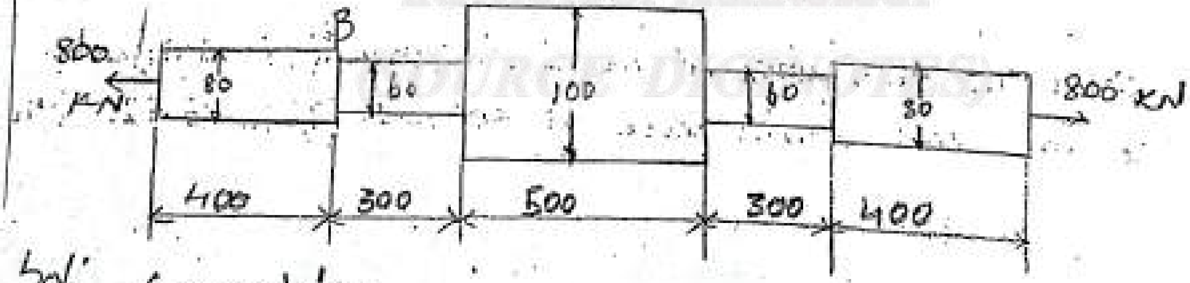
* Since the c/s of bar is uniform, the maximum stress will be in section CD, where the force is maximum

$$\therefore \sigma_{max} = \frac{60 \times 10^3 \times 4}{\pi \times (20 \times 10^{-3})^2} = 191 \times 10^6 \text{ N/m}^2$$

Stepped bars

- A circular bar of various c/s is subjected to a pull of 800 KN as shown. Determine the extension of the bar. $E = 204$ GPa

All dimensions are in mm



Sol: Given data

c/r bar of different cross-section

→ 2 length of 400 mm each of 80 mm diameter

→ 2 section length of 300 mm each of 60 mm dia

→ A section of length 500 mm & diameter is 100 mm

→ Load = 800 KN, $E = 204$ GPa

source: diginotes.in

To find : Total Extention of bar

w.k.t, Total extention = $\delta L = \frac{PL}{AE}$

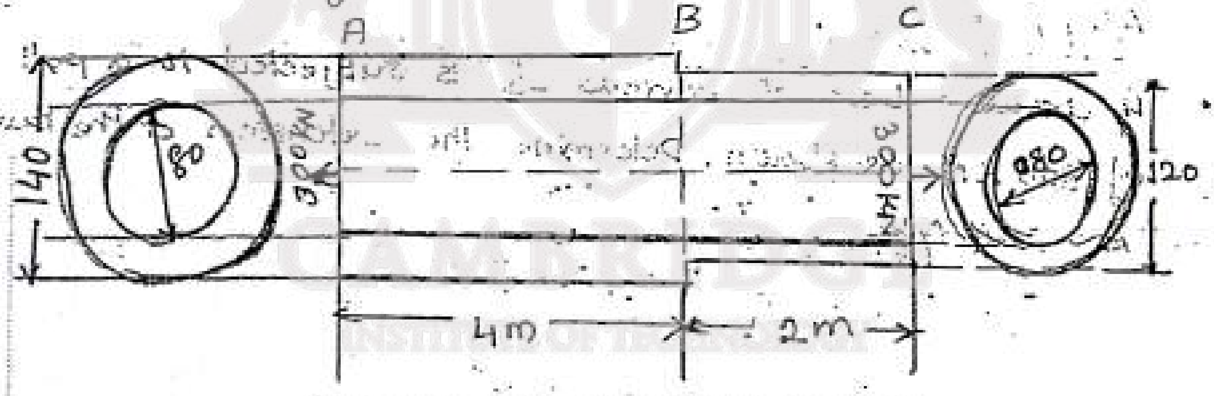
But here L & A's are different

$\therefore \delta L = \frac{P}{E}\left(\frac{2L_1}{A_1} + \frac{L_2 \times 2}{A_2} + \frac{L_3}{A_3}\right)$

$= \frac{800 \times 10^3}{204 \times 10^9} \times \frac{4}{\pi}\left[\frac{2 \times 400}{80^2} + \frac{2 \times 300}{60^2} + \frac{500}{100^2}\right]$

$\delta L = 1.708$ mm

A 6m long hollow bar of circular section has 140 mm diameter for a length of 4m, while it has 120 mm diameter for a length of 2m. The bore diameter is 80 mm throughout as shown in fig.



Find the elongation of the bar, when it is subjected to an axial tensile force of 300 kN. Take E = 200 GPa.

Sol. Given data

Segment AB

Diameter of AB = D_1 = 140 mm

Inner diameter = d = 80 mm

Length of AB = 4 m = 4000 mm

Load = 300 kN

Segment BC

Outer dia of BC = D_2 = 120 mm

Inner dia of BC = d = 80 mm

Length of BC = 2×10^3 mm

Load = 300 kN

source: diginotes.in

$E = 200\,GPa = 200 \times 10^3\ N/mm^2$

Area of AB = $A_1 = \frac{\pi}{4}\left[D_1^2 - d^2\right]$

$= \frac{\pi}{4}\left[140^2 - 80^2\right] =$ mm^2

Area of BC = $A_2 = \frac{\pi}{4}\left[D_2^2 - d^2\right]$

$= \frac{\pi}{4}\left[120^2 - 80^2\right] =$ mm^2

∴ Total elongation of bar,

$\delta L = \delta L_1 + \delta L_2$

$= \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]$ [∵ P & E are same for both.]

$\delta L = 1.054\ mm$

- A compound bar ABC 1.5 m long is made up of 2 parts of alluminium & steel & the cross-sectional area of aluminium is twice that of the steel bar. The rod is subjected to an axial tensile load of 200 kN. If the elongation of aluminium & steel parts are equal, find the lengths of the 2 parts of the component bar. Take E for steel as 200 GPa, & E for alumenium as one-third of E for steel.

Sol Given data.

Total length $= L = 1.5\ m$

Let c/s area for aluminium $= A_a$

c/s " " steel $= A_s$

$A_a = A_s \times 2$

Axial tensile load $= P = 200\ kN$

E for steel $= E_s = 200 \times 10^3\ N/mm^2$

E for Al $= E_a = \frac{E_s}{3} = \frac{200}{3} \times 10^3\ N/mm^2$

source: diginotes.in

Let

length of steel bar = L_s

" " Al bar = L_a

elongation of steel bar = δL_s

" " Al " = δL_a

Total elongation = $\delta L = \delta L_s + \delta L_a$.

But given that $\delta L_s = \delta L_a$

$\Rightarrow \dfrac{P L_a}{A_a E_a} = \dfrac{P L_s}{A_s E_s}$

$\Rightarrow \dfrac{1.5 L_a}{A_s} = \dfrac{L_s}{A_s}$ $\quad \because A_a = 2A_s$

$\Rightarrow L_s = 1.5 L_a$

But Total length $L = L_s + L_a = 1.5 \times 10^3$

$\Rightarrow 1.5 \times 10^3 = 1.5 L_a + L_a = 2.5 L_a$

$\Rightarrow L_a = 600$ mm

$L_s = 900$ mm

Mechanics Of Materials 15ME34

Question Bank.

Module 1:

1) A circular rod of diameter 20mm & 500 mm long is subjected to a tensile force 45 KN. The modulus of elasticity for steel may be taken as 200 KN/mm². Find stress, strain & elongation of the bar due to applied load.

2) A hollow steel tube is to be used to carry an axial compressive load of 140 KN. The Yield stress for steel is 250 N/mm². A factor of safety of 1.75 is to be used in the design. The following 3 classes of tubes of external diameter 101.6 mm are available. Which section do you recommend?

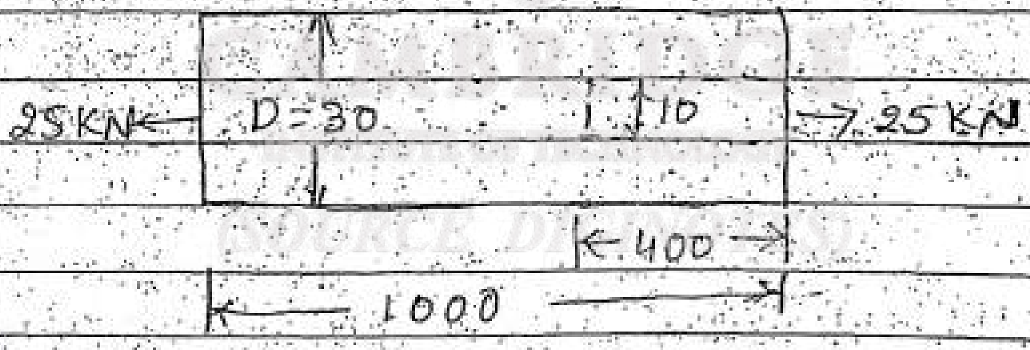
Class	Thickness
1) Light	3.65 mm
2) Medium	4.05 mm
3) Heavy	4.85 mm

3) A steel press has 4 tension members. Each member has a diameter of 16 mm. The largest load to be resisted by the press is to be 48 KN. Determine axial stress in the tension members.

source: diginotes.in

(4) A compound bar is made of solid bronze bar 0.6 m long & steel tube connected in series. The outer diameter of steel portion & diameter of bronze portion are equal to 35mm. The compound bar is subjected to a tensile force of 84 kN. Allowable stress of steel tube is 130 MPa. Taking $E_b = 85$ GPa & $E_s = 210$ GPa, determine
(i) Inner diameter of steel tube.
(ii) Length of steel portion such that deformation in steel tube is 1.5 times that of bronze bar.

(5) A bar of length 1000 mm & dia 30 mm is centrally bored for 400 mm length, the bore dia being 10 mm as shown. Under a load of 25 kN, if the extension of the bar is 0.18 mm, what is the modulus of elasticity of the bar?

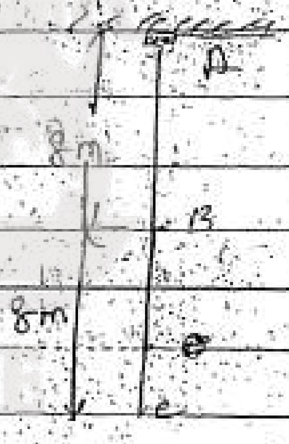


6) A hollow steel tube 3.5 m long has external diameter of 120 mm. In order to determine the internal diameter, the tube was subjected to a tensile load of 400 kN & extension was measured to be 2mm. If the E for the tube

material is 200 GPa, determine the internal diameter of the tube.

07) An alloy wire of 2 mm² c/s area & 12 N weight hangs freely under its own weight. Find the max length of the wire, if its extension is not be exceed 0.6 mm. Take E for the wire material as 150 GPa.

8) A steel wire ABC 16m long having c/s area of 4 mm² weighs 20 N as shown in fig. If the modulus of elasticity for the material is 200 GPa, find the deflections @ C & B.

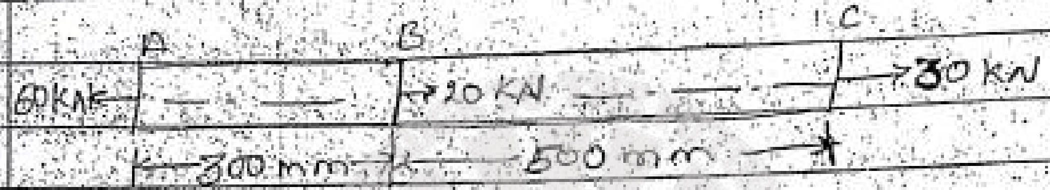


9. A steel bar 2m long & 40 mm in diameter is subjected to an axial pull of 80 kN. Find the length of the 20 mm diameter bore, which should be centrally carried out, so that the total elongation should increase by 20% under the same pull. Take E = 200 GPa.

source: diginotes.in

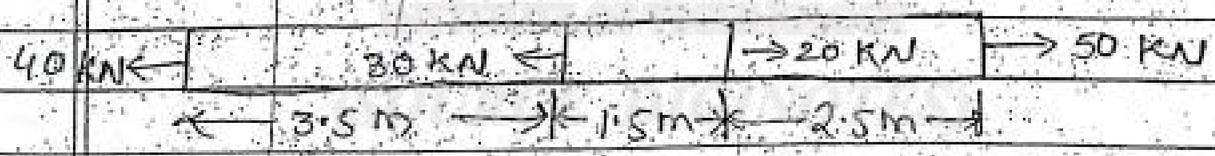
Uniform bar subjected to loads along length.

1) A steel bar of c/s area 200mm² is loaded as shown. Find the change in length of the bar. Take E as 200 GPa.

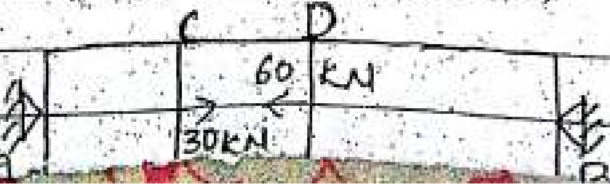


2) A brass bar, having c/s area of 500mm² is subjected to axial forces as shown below. Find total elongation of bar. Take E = 80 GPa

3) A copper rod ABCD of 800 mm² c/s area & 7.5 m long is subjected to forces as shown. Find the total elongation of the bar. Take E = 100 GPa



4) A bar of 800 mm length is attached rigidly @ A & B as shown in fig. If E = 200 MPa, determine reactions @ the 2 ends. If the bar diameter is 25 mm, find the stresses & change in length of each portion.



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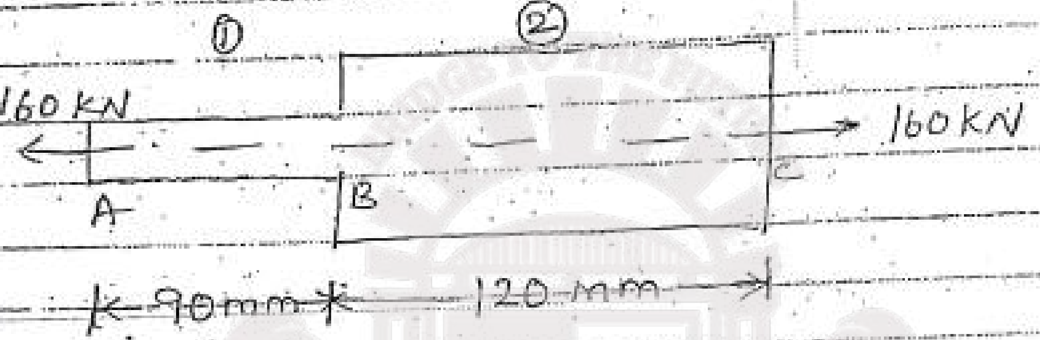
Stress & deformation in a bar [stepped]

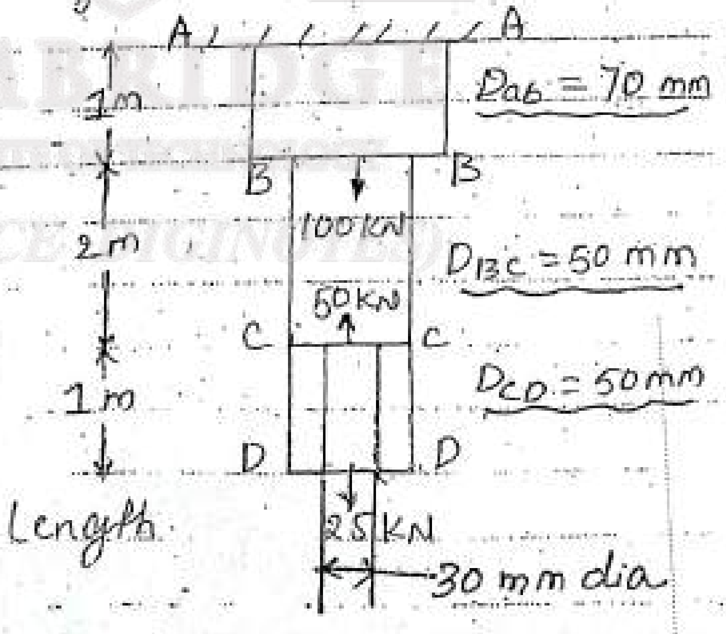
1) An automobile component shown in fig. is subjected to an tensile load of 160 kN



$A_1 = 50\ mm^2$ $A_2 = 100\ mm^2$.

Determine the total elongation of the composed if its modulus of elasticity is 200 GPa.

2) A circular rod ABCD of different c/s is loaded as shown below. Find the maximum stress induced in the rod, & its δL. Take E = 200 GPa.



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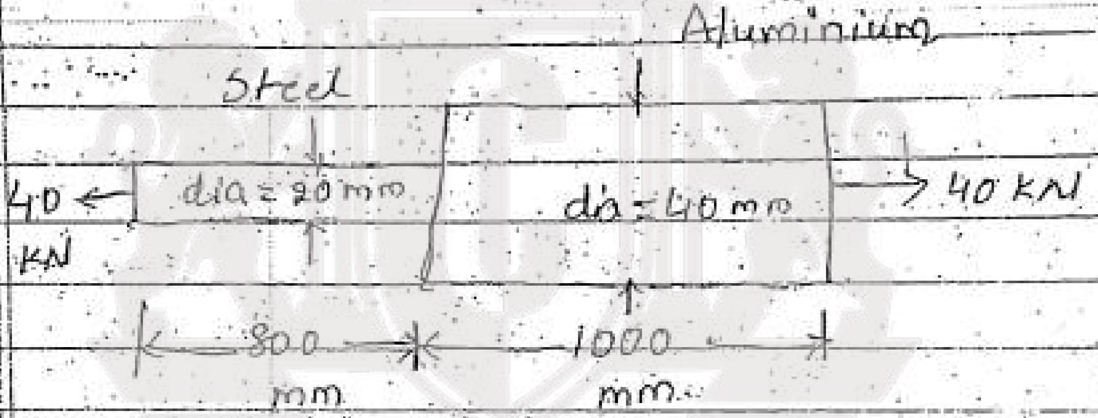
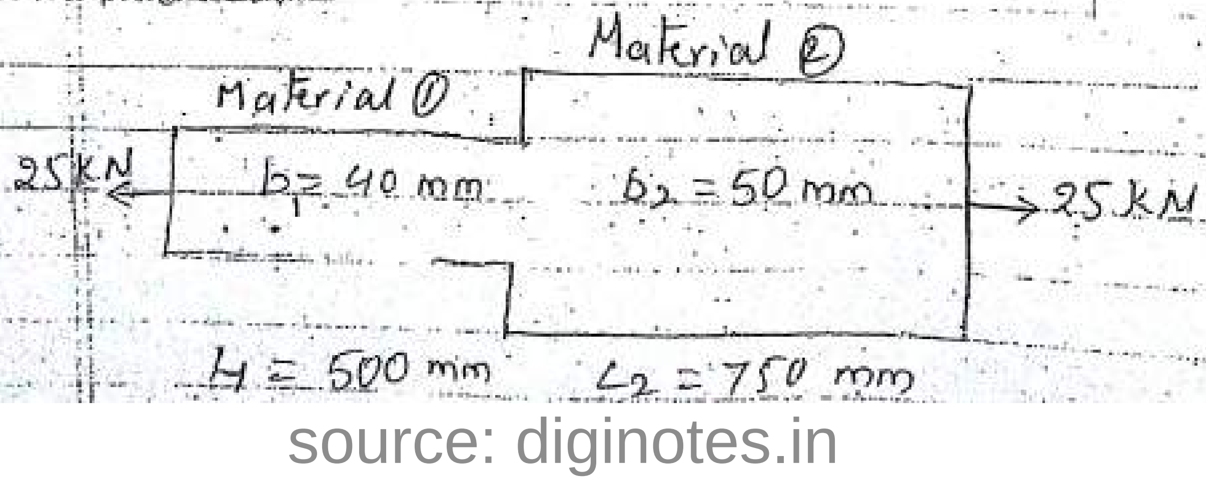
Stress in a bar of different materials.

1. A steel rod, 20 mm diameter & 800 mm long, is rigidly attached to an aluminium rod, 40 mm in dia & 1m long as shown below. The combination is subjected to a tensile load of 40 kN. Find the stress in the material & the total elongation of the bar. Take E for stee = $E_s = 200$ GPa, $E_{al} = 70$ GPa



2. The stepped bar as shown, is made up of 2 different materials. The material one has $E_1 = 2\times10^5$ N/mm², while other $E_2 = 1\times10^5$ N/mm². Find the extension of the bar under a pull of 25 KN if both the portions are 20 mm in thickness.



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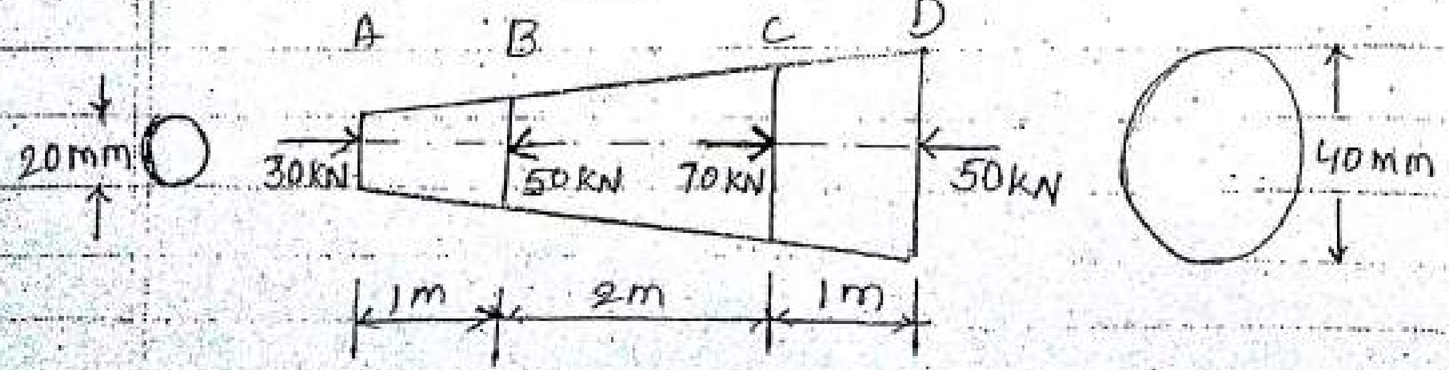
Tapered bars [& combination]

1. A circular alloy bar 2m long uniformly tapers from 30 mm dia to 20 mm diameter. Calculate the elongation of the rod under an axial force of 50 kN. Take E for the alloy as 140 GPa.

2. A steel flat of thickness 10 mm tapers from 60 mm @ one end to 40 mm @ other end in a length of 600 mm. If the bar is subjected to a load of 60 kN, find its extension. Take $E = 2 \times 10^5$ MPa. What is the % error if average area is used for calculating extension?

3. Two circular bars A & B of the same material are subjected to the same pull (P) & are deformed by the same amount. What is the ratio of their lengths, if one of them has a constant diameter of 60 mm & the other uniformly tapers from 80 mm from one end to 40 mm @ the other.

4) For a given tapered alloy, find δL. Take $E = 120$ GPa.



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5. An alloy bar of 1m length has square section throughout, which tapers from one end of 10 mm x 10 mm to the other end of 20 mm x 20 mm. Find the changes in its length due to an axial tensile load of 80 kN. Take E for alloy = 120 GPa

6. A steel plate of 20 mm thickness tapers uniformly from 100 mm to 50 mm in a length of 400 mm. What is elongation of the plate, if an axial force of 80 kN acts on it? Take E = 200 GPa

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Krishna Xerox 9008281471BU

MODULE - 2

COMPOUND STRESSES:

Introduction, plane stress, stresses on principal stresses and maximum shear ..., Mohr's circle for plane stress (7 Hours)

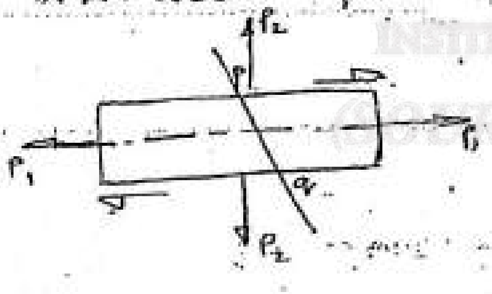
Plane ... *

Introduction :-

* We have considered only normal stresses acting on cross- sections, such as c/s mN of a bar AB, when a bar is subjected to either tensile force or compressive force acting along the axis of the bar.

* In compound stresses we shall analyze the stresses induced on inclined sections such as pq.

* In actual practice, all 3 types of stresses i.e, tension, compression and shear stress may act simultaneously on appropriate planes passing through a point in the strained material



* Therefore analysis of such a general stress system and the resultant stress on critical inclined planes which may carry greater stresses than applied is necessary.

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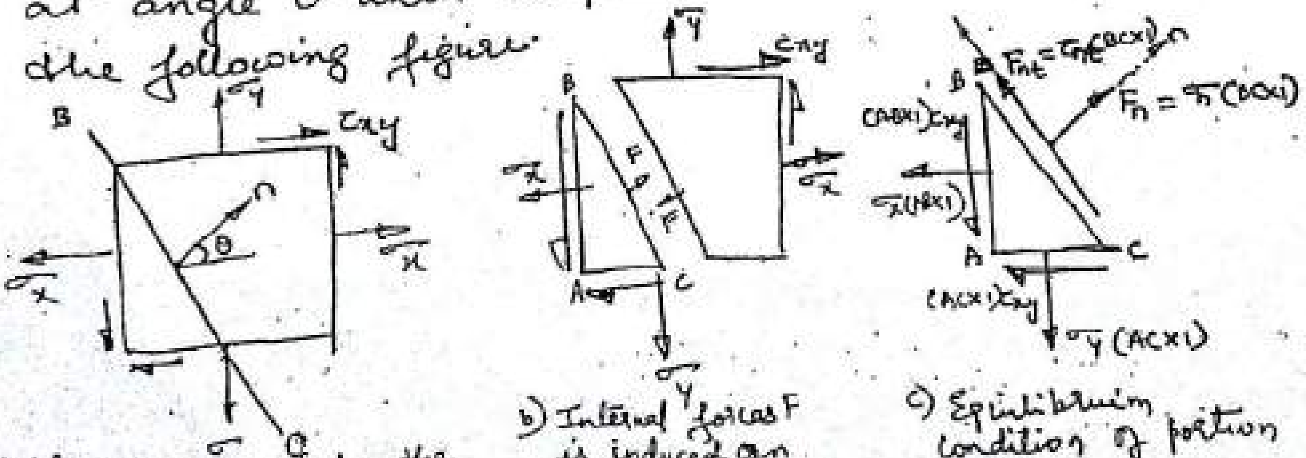
Stress system:-

A plane stress system involves a point in a structural or a machine member being subjected to the stresses in a single plane.

* Axially loaded bar, a shaft subjected to torsion, a beam subjected to bending moment will be under plane stress system, which involves the points in the member being subjected to the stresses in a single plane i.e., the stresses are induced along x & y directions and no stresses are indicated in z-direction.

* A point consists of infinite number of planes oriented in different directions and in which each plane is subjected to either normal stress or shear stress, or both stresses of certain magnitude depending upon its orientation and the stress system acting on a point.

* Consider an element in a body of unit thickness, subjected to a plane stress system consisting of 2 mutual perpendicular normal stresses σ_x and σ_y and shear stress τ_{xy}. The following procedure is adopted to find the stresses acting on a arbitrary plane, say BC whose normal is oriented at angle θ with respect to x-axis as shown in the following figure.



a) Divide the element along the plane BC on which stresses are

b) Internal forces F is induced on plane BC.

c) Equilibrium condition of portion ABC.

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i) Divide the element into 2 portions along the plane BC on which the stresses are to be found. The two portions are subjected to the internal resisting forces F, equal in magnitude and opposite in direction on their contact planes as shown in fig (a) in order to keep the element in equilibrium condition.

ii) Consider the portion ABC of the element and resolve the forces F into two mutually ⊥r components F_n and F_{nt} which are ⊥r and ||el resp. to the plane BC and whose magnitudes and direction are to be found.

iii) The portion ABC will be in equilibrium conditions under the influence of the forces which are the products of the stresses and the corresponding plane areas on which they act as shown in figure (c). Resolve all the applied forces $\sigma_x \times (AB \times 1)$, $\sigma_y \times (AC \times 1)$, $\tau_{xy} \times (AB \times 1)$ and $\tau_{yx} \times (AC \times 1)$ along n and t axes.

iv) Obtain the equilibrium equations $\Sigma F_n = 0$ and $\Sigma F_{nt} = 0$ for 2 mutually perpendicular directions i.e, along n and t axes. Solving these 2 equations, we can find the magnitude and directions of normal stress σ_n and shear stress τ_n induced in the plane.

* Dimensions of a structural or a m/c member are generally found such that maximum stresses induced should not exceed the allowable stresses for the material of the member. Therefore it is important that we find the maximum stress induced in a member subjected to plane certain plane stress system and the magnitudes of which are found based on the syste stresses induced on arbitrary plane.

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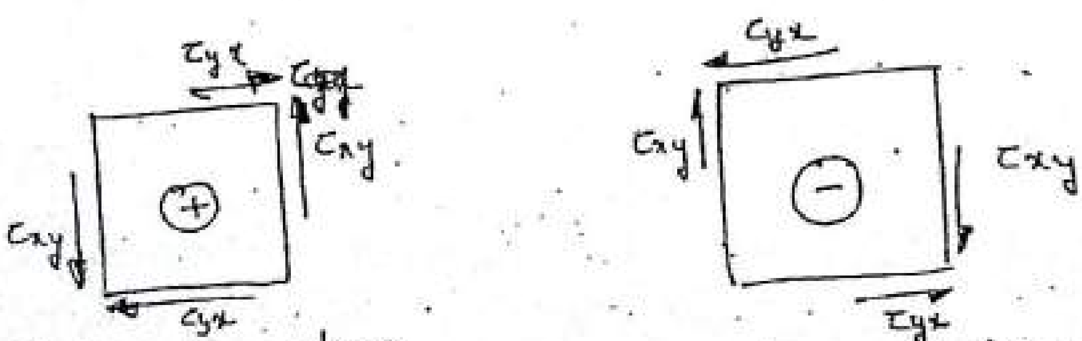
* We can obtain the magnitudes of maximum normal stress and maximum shear stress and the orientation of the corresponding planes by following 2 important steps :-

(i) Differentiate the equations related to normal stress and shear stress induced on an arbitrary plane with respect to its orientation θ and equate the resulting equations to zero. We can obtain the orientations θ_P and θ_S of the plane subjected to maximum normal stress and maximum shear stress respectively from the 2 equations.

(ii) Substituting the orientation θ_P of the maximum normal stress plane and orientation θ_S of the maximum shear stress plane into the equations for normal stress and shear stress on an arbitrary plane respectively, we can obtain the magnitudes of maximum normal stress and maximum shear stress.

Sign Convention :-

(a) Positive normal stress (b) negative normal stress



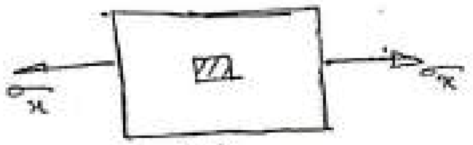
Positive shear stress Negative shear stress

$\tau_{xy} = \tau_{yx}$

source: diginotes.in

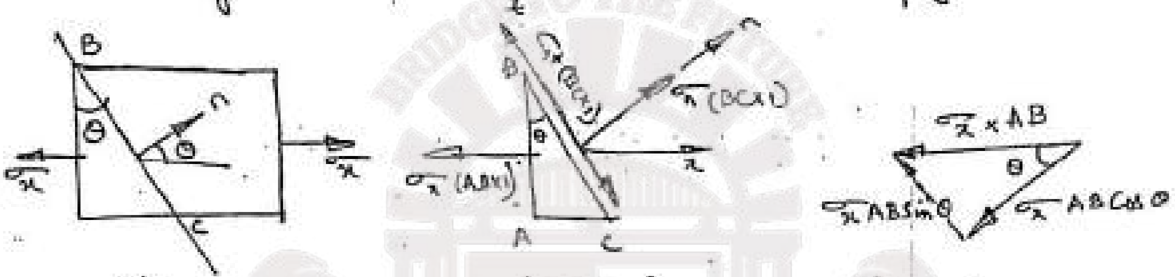
Krishna Xerox 9008...

Point under uniaxial ~~stress~~ direct stress :-



(a)

* A uniform bar of a unit thickness subjected to uniaxial stress σ_x is shown in figure (a). A point is represented by a small rectangular element subjected to a stress σ_x is considered for the stress analysis and an enlarged view is as shown in figure (b)



(b) (c) (d)

* Figure (c) shows the forces acting on the element which are obtained by multiplying the stresses and the corresponding plane areas. The oblique plane BC is subjected to normal force σ_n (BC×1) and shear force τ_{nt} (BC×1)

* **Stresses on a arbitrary plane**

→ The normal stress σ_n and shear stress τ_{nt} on the plane BC may be obtained by solving equilibrium equations for n and t directions shown in figure C.

→ Forces due to stress σ_x is resolved along n and t directions as shown in figure (d).

→ For Equilibrium of forces on element ABC, $\Sigma F_n = 0$ and $\Sigma F_t =$

$\Sigma F_n = 0$

$\sigma_n (BC \times 1) - \sigma_x AB \cos\theta = 0$

$\therefore \sigma_n = \frac{AB}{BC} \cos\theta \, \sigma_x$

or $\sigma_n = \sigma_x \cos\theta \cos\theta$ $\quad \left[as \ \frac{AB}{AC} = \cos\theta \right]$

$$\therefore \boxed{\sigma_n = \sigma_x \cos^2\theta} \quad \text{——— (1)}$$

source: diginotes.in

$\Sigma F_t = 0$

$\tau_{nt}(BC \times 1) - \sigma_x AB \sin\theta = 0$

or $\tau_{nt} = \sigma_x \frac{AB}{BC} \sin\theta$

$= \sigma_x \cos\theta \sin\theta$ $\quad$ [as $\frac{AB}{BC} = \cos\theta$]

$\therefore$ $\boxed{\tau_{nt} = \frac{\sigma_x}{2} \sin 2\theta}$ —— (2) $\quad$ [as $\sin 2\theta = 2\sin\theta\cos\theta$ or $\cos\theta\sin\theta = \frac{\sin 2\theta}{2}$]

* <u>Maximum</u> <u>and</u> <u>minimum</u> <u>stress</u> :-

I) From equation of normal stress $\sigma_n = \sigma_x \cos^2\theta$.
We can see that the magnitude of normal stress will be maximum when $\cos^2\theta = 1$ and minimum when $\cos^2\theta = 0$ respectively.

$\therefore$ Orientation of planes carrying maximum and minimum normal stress are $\theta_{P1} = 0$ and $\theta_{P2} = 90°$, which are the planes pender perpendicular and parallel to the axis of the member respectively as shown in the following figure.

$\therefore$ Maximum normal stress $\sigma_1 = \sigma_x \cos^2 0 = \sigma_x$ ... [as $\theta_{P1} = 0$]

and

Minimum normal stress $\sigma_2 = \sigma_x \cos^2 90 = 0$ [as $\theta_{P2} = 90$].

II] From equation of <u>shear stress</u> on an arbitrary plane is given by $\tau_{nt} = \frac{\sigma_x}{2} \sin 2\theta$

The shear stress will be maximum when $\sin 2\theta = 1$ i.e. when $\theta_{S1} = 45°$

Substituting the value of θ_{S1} into the equation of τ_{nt}, we get,

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Maximum shear stress $\tau_1 = \frac{\sigma_2}{2}$

or $\tau_1 = \frac{\sigma_1}{2}$

$\therefore$ maximum shear stress induced in a Body subjected to uniaxial stress is half the maximum normal stress.

Pb) A uniform bar is subjected to an axial tensile stress of 100 N/mm². Determine,

i) stresses acting on a plane which is at an angle of 60° with reference to the 100 N/mm² stress plane.

ii) Magnitude of maximum and minimum stresses induced and the position of their planes.

iii) Magnitude of normal stress on the plane of maximum shear stress and magnitude of shear stress on the plane of maximum normal stress

Solution Given tensile stress $\sigma_x = 100\ N/mm^2$

i) Stresses on a plane at $\theta = 60°$

a) Normal stress $\sigma_n = \sigma_x \cos^2\theta = 100 \times \cos^2 60$

$\therefore \sigma_n = 25\ N/mm^2$

b) Shear stress. $\tau_{nt} = \frac{\sigma_x}{2} \sin 2\theta = \frac{100}{2} \sin(2 \times 60)$

$= 43.30\ N/mm^2$

(ii) Maximum and minimum stresses

a) Maximum normal stress $\sigma_1 = \sigma_x = 100\ N/mm^2$

Minimum normal stress $\sigma_2 = 0$

Maximum normal stress σ_1 acts on a plane which is perpendicular to the axis of the body i.e. $\theta_{P_1} = 0°$.

Minimum normal stress σ_2 is zero on the planes which are parallel to the axis $\theta_{P_2} = 90°$.

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source: diginotes.in

b) Maximum and minimum shear stress $\tau_{12} = \pm \frac{\sigma_x}{2}$.

$$\tau_{12} = \pm \frac{100}{2} = \pm 50 \text{ N/mm}^2.$$

Orientation of maximum and minimum shear stress planes $\theta_{s1} = 45^\circ$ and $\theta_{s2} = 135^\circ$.

iii) a) Magnitude of normal stress on the plane of maximum shear stress (i.e. $\theta = 45^\circ$, $\sigma = ?$)

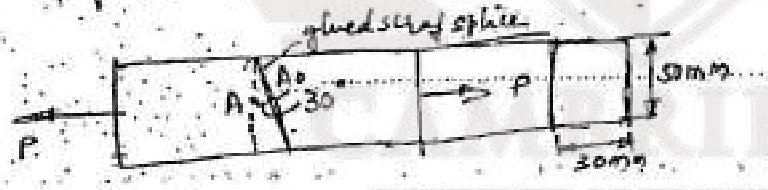
$$\sigma_n = \sigma_x \cos^2\theta = 100 \cos^2 45 = 50 \text{ N/mm}^2$$

b) Magnitude of shear stress on the plane of maximum normal stress ($\tau = ?$ at $\theta = 0^\circ$)

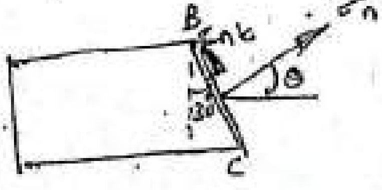
$$\tau = \frac{\sigma_x}{2} \sin 2\theta = \frac{100}{2} \sin(2 \times 0)$$

$$\tau = 0.$$

Pb. 2] Two uniform bar with rectangular c/s 30mm X 50mm are joined by a simple scarf splice as shown in the figure. The allowable stresses of glued splice are 20 MPa in tension and 9 MPa in shear respectively. Determine the largest axial force P that can be applied on the member, so that there is no failure of the joint.



Sol^n:



Given

Area $A = 30 \text{ mm} \times 50 \text{ mm}$

$\sigma_{yd} = 20 \text{ MPa}$

$\tau_{yd} = 9 \text{ MPa}$

$\theta = 30^\circ$

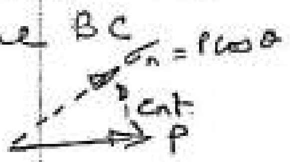
source: diginotes.in

i) Resisting normal force on the plane BC

$R_n = A_\theta \sigma_{yd}$ where A_θ is the area of the inclined plane at 30° to the axis.

where $A_\theta = \frac{A}{\cos\theta}$

Resisting normal force on plate = Component of the applied force normal to plane BC



$$\frac{A}{\cos\theta}\sigma_{yd} = P\cos\theta$$

$$\frac{30\times50}{\cos 30^\circ}\times 20 = P\cos 30^\circ$$

or $P = 40{,}000\,N = 40\,kN$

ii) Resisting shear force on the plane $R_\tau = A_\theta \tau_{yd}$

Resisting ~~normal~~ shear force on the plate BC = Component of applied force ~~no~~ parallel to plane BC

$$\tau_{yd}\frac{A}{\cos\theta} = P\sin\theta$$

$$9\times\frac{30\times50}{\cos 30} = P\sin 30$$

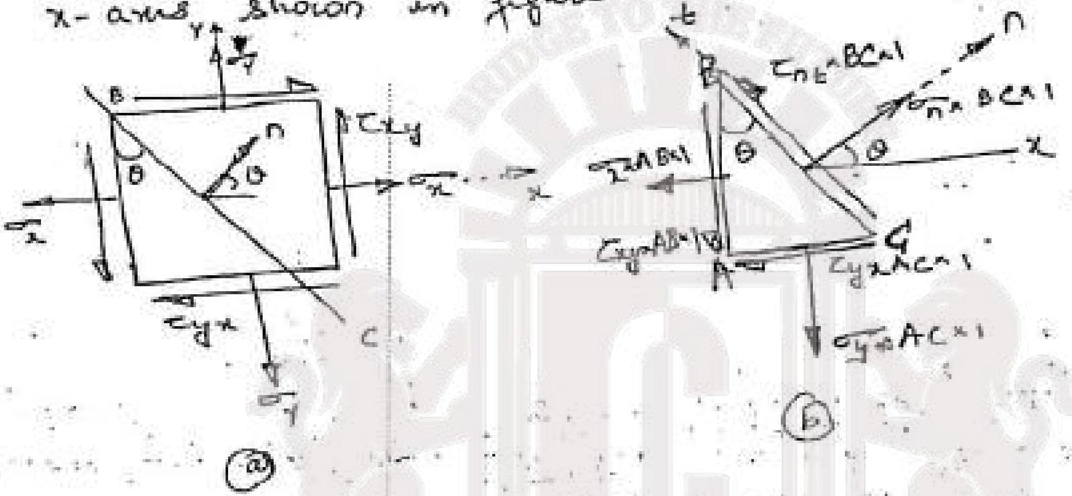
$\therefore P = 31176.91$ or $31.18\,kN$

Hence the maximum force $P = 31.18\,kN$ may be applied on the member, for the joint to be safe.

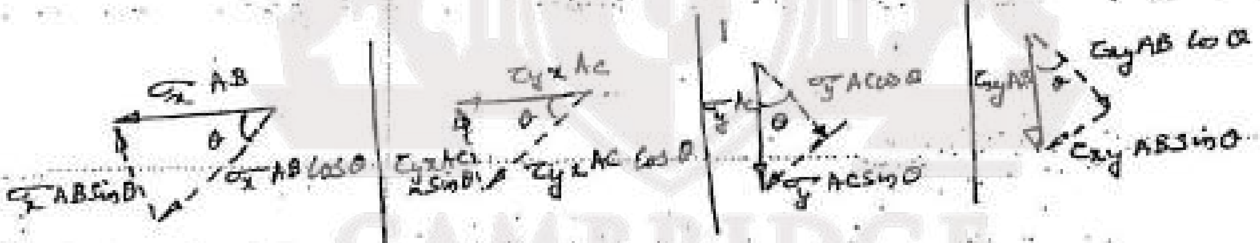
source: diginotes.in

Point subjected to General stress system

* Consider the most general case of stress system where an element of unit thickness is subjected to the mutually ⊥r stresses, σ_x, σ_y and σ_z shear stress τ_{xy} as shown in figure a. The forces acting on one of the portions of the element formed by passing a plane BC whose normal is at an angle θ w.r.t x-axis, shown in figure b.



ⓐ ⓑ



* stresses on a given plane
* For equilibrium of forces on the element in n direction

$\Sigma F_n = 0$

$$\sigma_n \times (BC \times 1) - \sigma_x AB\cos\theta - \tau_{yx} AC\cos\theta - \sigma_y AC\sin\theta - \tau_{xy} AB\sin\theta = 0$$

$$\sigma_n = \sigma_x \frac{AB}{BC}\cos\theta + \tau_{yx}\frac{AC}{BC}\cos\theta + \sigma_y \frac{AC}{BC}\sin\theta + \tau_{xy}\frac{AB}{BC}\sin\theta$$

From figure ⓑ $\frac{AB}{BC} = \cos\theta$ and $\frac{AC}{BC} = \sin\theta$.

substituting these values in equation of σ_n, we get

source: diginotes.in

$\sigma_n = \sigma_x \cos^2\theta + \tau_{yx} \sin\theta \cos\theta + \sigma_y \sin^2\theta + \tau_{xy} \cos\theta \sin\theta$

$= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{yx} \sin\theta \cos\theta + \tau_{xy} \sin\theta \cos\theta$.

Substitute $\cos^2\theta = \frac{1+\cos 2\theta}{2}$, $\sin^2\theta = \frac{1-\cos 2\theta}{2}$, $\sin\theta\cos\theta = \frac{\sin 2\theta}{2}$

in above equation we get

$\sigma_n = \sigma_x \left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y \left(\frac{1-\cos 2\theta}{2}\right) + \tau_{yx} \frac{\sin 2\theta}{2} + \tau_{xy} \frac{\sin 2\theta}{2}$

$= \sigma_x \left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y \left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy} \sin 2\theta$

[since $\tau_{xy} = \tau_{yx}$]

$\therefore \sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$ —— (1)

→ For equilibrium forces on the element in t direction

$\Sigma F_t = 0$

$\tau_{nt} (BC \times 1) + \sigma_x AB \sin\theta + \tau_{yx} AC \sin\theta - \sigma_y AC \cos\theta - \tau_{xy} AB \cos\theta =$

$\tau_{nt} = -\sigma_x \frac{AB}{BC} \sin\theta - \tau_{yx} \frac{AC}{BC} \sin\theta + \sigma_y \frac{AC}{BC} \cos\theta + \tau_{xy} \frac{AB}{BC} \cos\theta$

$\therefore \tau_{nt} = -\sigma_x \cos\theta \sin\theta - \tau_{yx} \sin^2\theta + \sigma_y \sin\theta \cos\theta + \tau_{xy} \cos^2\theta$

$= (\sigma_y - \sigma_x) \cos\theta \sin\theta - \tau_{xy} \sin^2\theta + \tau_{xy} \cos^2\theta$

$= \frac{(\sigma_y - \sigma_x)}{2} \sin 2\theta - \tau_{xy} \left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy} \left(\frac{1+\cos 2\theta}{2}\right)$

$= \left(\frac{\sigma_y - \sigma_x}{2}\right) \sin 2\theta + \frac{2 \cos 2\theta}{2} \tau_{xy}$

$\tau_{nt} = \left(\frac{\sigma_y - \sigma_x}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$ —— (2)

* substituting the value of θ the inclination of normal to a given plane with respect to x-axis in equations (1) and (2) we get normal stress and shear stress acting on the plane

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Principal Stresses

* Maximum and minimum normal stresses induced in a body are known as principal stresses.
* A plane subjected to maximum or minimum normal stresses and which is not subjected to any shear stress is known as principal plane.
* The plane carrying maximum and minimum normal stresses are known as major principal plane and minor principal planes respectively.
* Differentiate the equation σ_n with respect to θ and equate it to zero to get the orientation of principal planes.

$$\frac{d}{d\theta}\left[\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos 2\theta+\tau_{xy}\sin 2\theta\right]=0,$$ where the stresses σ_x, σ_y and τ_{xy} are constants.

$$\left[0-2\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta+2\tau_{xy}\cos 2\theta\right]=0$$

or $$\tan 2\theta=\frac{2\tau_{xy}}{\sigma_x-\sigma_y} \quad \text{--- (3)}$$

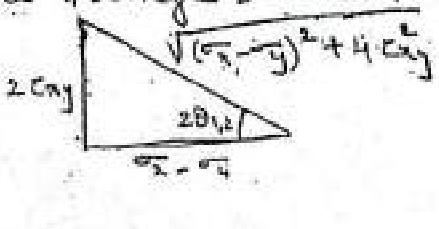
From the above equation, the orientation of major principal plane and minor principal planes are given by,

$$\theta_{P_1}=\frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right]$$

and $$\theta_{P_2}=\theta_{P_1}+90^\circ.$$

The angle between major and minor principal planes is always 90°.

* Now draw a triangle based on equation (3)



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To get the magnitudes of maximum and minimum normal stresses:- From the triangle we have

$$\sin 2\theta_{P1,2} = \pm \frac{2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

and

$$\cos 2\theta_{P1,2} = \pm \frac{\sigma_x-\sigma_y}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

Now according to equation ①,

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta_{P1,2} + \tau_{xy}\sin 2\theta_{P1,2}$$

$$= \frac{\sigma_x+\sigma_y}{2} \pm \frac{\sigma_x-\sigma_y}{2} \cdot \frac{(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} \pm \tau_{xy}\frac{2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

$$= \frac{\sigma_x+\sigma_y}{2} \pm \frac{1}{2\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}\left[(\sigma_x-\sigma_y)^2+4\tau_{xy}^2\right]$$

or

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \frac{1}{2}\left[(\sigma_x-\sigma_y)^2+4\tau_{xy}^2\right]^{1-\frac{1}{2}}$$

$$= \frac{\sigma_x+\sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$$

$$= \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\frac{(\sigma_x-\sigma_y)^2}{4}+\frac{4}{4}\tau_{xy}^2}$$

or

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}.$$

The magnitudes of major principal stress σ_1 (or maximum normal stress) and minor principal stress σ_2 (or minimum normal stress) are obtained by considering plus and minus signs respectively in the above equation.

source: diginotes.in

Maximum and minimum shear stress :-

* The maximum and minimum shear stress induced in the element are obtained by differentiating the equation for τ_{nt} with respect to θ and equating it to zero.
* We have, equation of shear stress acting tangential to the element,

$$\tau_{nt} = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

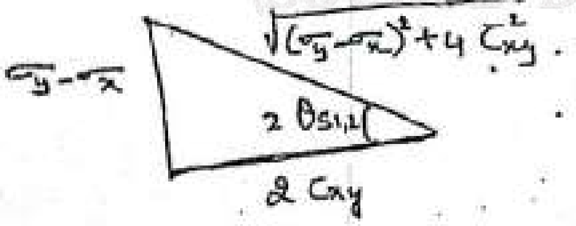
differentiating the above equation with respect to 'θ', we get,

$$\frac{d}{d\theta}\left[\left(\frac{\sigma_y - \sigma_x}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta\right] = 0$$

$$2\left(\frac{\sigma_y - \sigma_x}{2}\right)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

or $$\tan 2\theta_{s1,2} = \frac{\sigma_y - \sigma_x}{2\tau_{xy}} = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \quad \text{--- (4)}$$

* The values of θ_{s1} and $\theta_{s2} = \theta_{s1} + 90°$ calculated from the above formula give the orientations of maximum and minimum shear stress planes which are at the right angles to each other.
* Now draw a triangle based on eqn (4)



$$\sin 2\theta_{s1,2} = \pm\frac{\sigma_y - \sigma_x}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}} \quad \text{and} \quad \cos 2\theta_{s1,2} = \pm\frac{2\tau_{xy}}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}$$

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substituting these values in τ_{nt} equation we get

$$\tau_{nt} = \pm\left(\frac{\sigma_y - \sigma_x}{2}\right)\left[\frac{\sigma_y - \sigma_x}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}\right] \pm \tau_{xy}\left[\frac{2\tau_{xy}}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}\right]$$

$$= \pm \frac{\frac{(\sigma_y - \sigma_x)^2}{2}}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}} \pm \frac{2\tau_{xy}^2}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}$$

$$= \pm \frac{1}{2} \frac{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}$$

$$= \pm \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}$$

$$\therefore \tau_{1,2} = \pm\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

* From the above equation it can be seen that the maximum and minimum shear stress are equal in magnitude but opposite in sign.

* This is in conformation with the law of complimentary shear stress which states that the magnitudes of shear stresses acting on two mutually perpendicular planes are equal

i.e., $\tau_{xy} = \tau_{yx}$.

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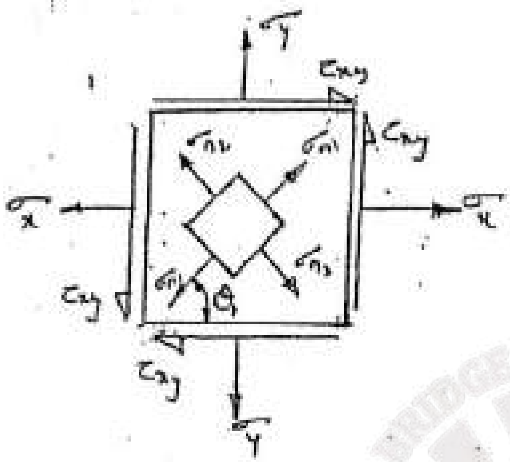
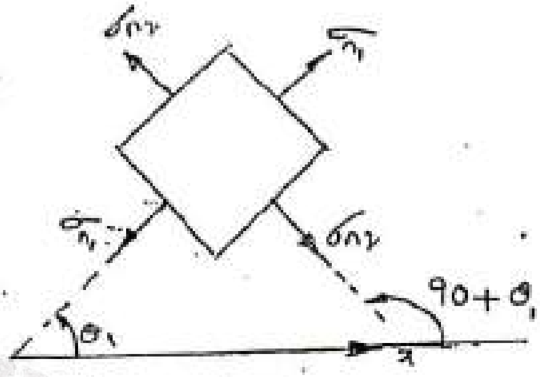
Prove that sum of any two orthogonal components of stress at a point is constant.





* A point is subjected to general stress system as shown in figure (a). Consider an element at the same point whose normals on its faces make the orientation θ_1 and $\theta_2 = \theta_1 + 90°$ with respect to x-axis.

We have to prove that

$$\sigma_x + \sigma_y = \sigma_{n_1} + \sigma_{n_2} \text{ at the same point}$$

* The equation for normal stress on an arbitrary plane,

$$\sigma_{n_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_1 + \tau_{xy}\sin 2\theta \quad \text{—— (1)}$$

Magnitude σ_{n_2} the normal stress on another face of the element is obtained by substituting $\theta_2 = \theta_1 + 90°$ in the above equation.

$$\sigma_{n_2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2(\theta_1 + 90) + \tau_{xy}\sin 2(\theta_1 + 90).$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_1 - \tau_{xy}\sin 2\theta_1 \quad \text{—— (2)}$$

Since,
$\cos 2(\theta + 90°) = -\cos 2\theta$
& $\sin 2(\theta + 90°) = -\sin 2\theta$

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Adding equation ① and ② we get

$$\sigma_{n_1} + \sigma_{n_2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_1 + \tau_{xy}\sin 2\theta_1$$
$$+ \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_1 - \tau_{xy}\sin 2\theta_1$$

$\sigma_{n_1} + \sigma_{n_2} = \sigma_x + \sigma_y$ Hence proved.

Prove that the normal stress acting on maximum and minimum shear stress planes is the average of 2 orthogonal components normal stresses acting on the point:-

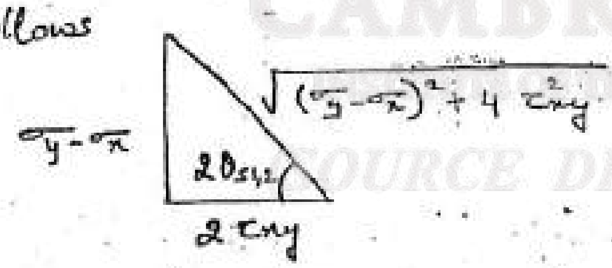
* The formula for normal stress acting on a plane is given by,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad (1)$$

* The orientation of maximum and minimum shear str is given by,

$$\tan 2\theta_{s_{1,2}} = \frac{\sigma_y - \sigma_x}{2\tau_{xy}}$$

* A triangle drawn on the above equation is as follows



Now, $\sin 2\theta = \frac{\sigma_y - \sigma_x}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}$ and $\cos 2\theta = \frac{2\tau_{xy}}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}$

Substituting these values in equation ① we get

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$$\sigma_{n_3} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\left(\frac{2\tau_{xy}}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}\right) + \tau_{xy}\left(\frac{\sigma_y - \sigma_x}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}\right)$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\tau_{xy}[\sigma_x - \sigma_y + \sigma_y - \sigma_x]}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}}$$

$$\therefore \sigma_{n_3} = \frac{\sigma_x + \sigma_y}{2} \quad \text{or} \quad \sigma_{n_3} = \frac{\sigma_1 + \sigma_2}{2}.$$

[because the sum of any two orthogonal components of stresses acting on a point is constant

ie $\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$].

* Therefore average of normal stress σ_{ave} acts on the planes of maximum and minimum shear stress.

<u>Prove that maximum shear stress is half of the difference of principal stress:</u>

Soln To prove that $\tau_1 = \frac{\sigma_1 - \sigma_2}{2}$

We have the equation for principal stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 - \sigma_2 = \left[\frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\right] - \left[\frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\right]$$

$$\therefore \sigma_1 - \sigma_2 = 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

But we have

$$\text{or} \quad \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{—— (1)}$$

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But we have $\tau_{1,2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

substituting this value in equation ① we get

$$\boxed{\tau_{1,2} = \frac{\sigma_1 - \sigma_2}{2}}$$

Pb3) A point in a body is subjected to tensile stresses of 100 N/mm² and 70 N/mm² along two mutually perpendicular directions. The point is also subjected to shear stress of magnitude 50 N/mm². Determine

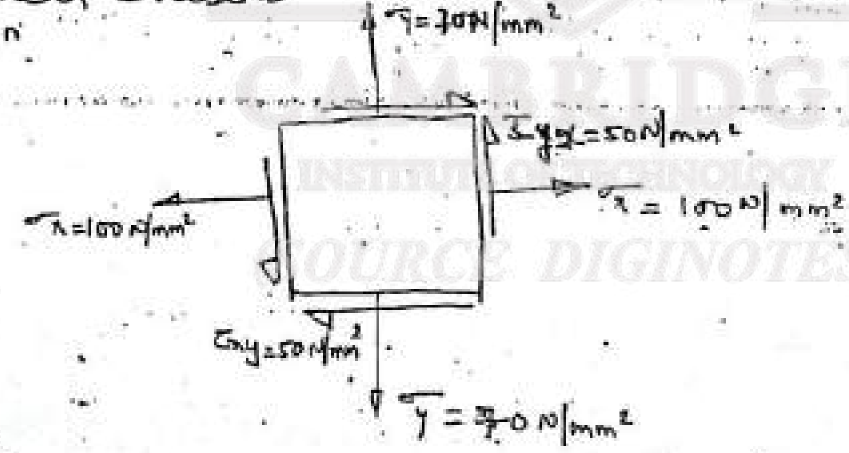
i) Normal stress and shear stress acting on a plane which is at an angle of 120° with reference to the 100 N/mm² stress plane.

ii) Magnitudes of principal stresses and maximum and minimum shear stress

iii) Orientations of principal planes and maximum and minimum shear stress planes.

iv) Normal stress on the planes of maximum and minimum shear stresses.

soln.



(i) Normal stress and shear stress at plane at 120°

We have

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \left(\frac{100 + 70}{2}\right) + \left(\frac{100 - 70}{2}\right)\cos(2\times 120) + 50\sin(2\times 120)$$

$$\therefore \sigma_n = 34.2 \ N/mm^2.$$

source: diginotes.in

But we have $\tau_{1,2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

substituting this value in equation ① we get

$$\boxed{\tau_{1,2} = \frac{\sigma_1 - \sigma_2}{2}}$$

Pb2) A point in a body is subjected to tensile stresses of $100\ N/mm^2$ and $70\ N/mm^2$ along two mutually perpendicular directions. The point is also subjected to shear stress of magnitude $50\ N/mm^2$. Determine.

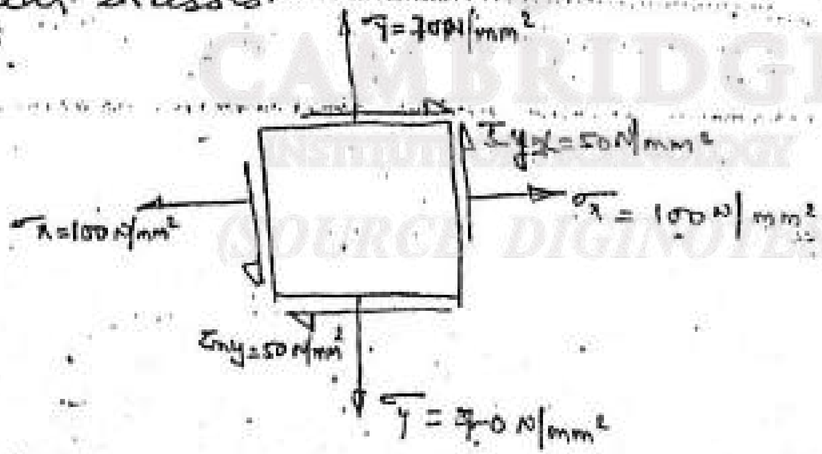
i) Normal stress and shear stress acting on a plane which is at an angle of 120° with reference to the $100\ N/mm^2$ stress plane.

ii) Magnitudes of principal stresses and maximum and minimum shear stress

iii) Orientations of principal planes and maximum and minimum shear stress planes.

iv) Normal stress on the planes of maximum and minimum shear stresses.

Sol^n.



(i) Normal stress and shear stress at plane at 120°

We have

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \left(\frac{100 + 70}{2}\right) + \left(\frac{100 - 70}{2}\right)\cos(2\times120) + 50\sin(2\times120)$$

$$\therefore \sigma_n = 34.2\ N/mm^2$$

$$\tau_{nt} = \left(\frac{\sigma_y - \sigma_x}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \left(\frac{70-100}{2}\right)\sin(2\times 120) + 50\times\cos(120\times 2)$$

$$\tau_{nt} = -12.01\ N/mm^2$$

(ii) Magnitudes of σ_1, σ_2 and τ_{max}, τ_{min}.

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{100+70}{2} + \sqrt{\left(\frac{100-70}{2}\right)^2 + 50^2} \quad \Big| \quad \sigma_2 = \frac{100+70}{2} - \sqrt{\left(\frac{100-70}{2}\right)^2 + 50^2}$$

$$\sigma_1 = 137.2\ N/mm^2 \qquad \sigma_2 = 32.8\ N/mm^2$$

$$\tau_{1,2} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\sqrt{\left(\frac{100-70}{2}\right)^2 + 50^2} = \pm 52.2\ N/mm^2$$

$$\tau_1 = 52.2\ N/mm^2 \quad \tau_2 = -52.2\ N/mm^2$$

(iii) Orientations of maximum and minimum stress planes ($\theta_{P_{1,2}}$)

Orientations of principal plane

$$\theta_{P_{1,2}} = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right] = \frac{1}{2}\tan^{-1}\left[\frac{2\times 50}{100-70}\right]$$

$$\theta_{P_1} = 36.65^\circ$$

and $\theta_{P_2} = 36.65^\circ + 90^\circ = 126.65^\circ$

(iv) Normal stress on the planes of maximum and minimum shear stress:-

First orientations of maximum and minimum shear stress have to be found out.

$$\theta_{S_1} = \frac{1}{2}\tan^{-1}\left[\frac{\sigma_y-\sigma_x}{2\tau_{xy}}\right] = \frac{1}{2}\tan^{-1}\left[\frac{70-100}{2\times 50}\right] = \frac{1}{2}\tan^{-1}[0.6]$$

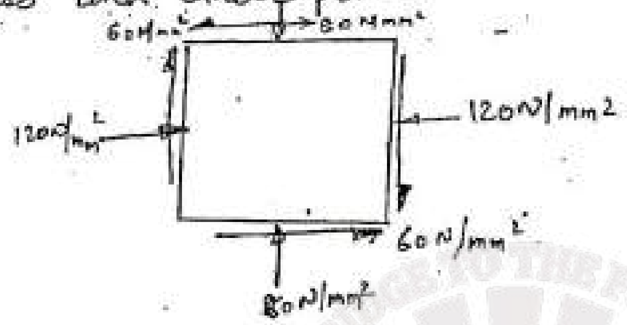
$$\theta_{S_1} = -8.35^\circ \text{ and } \tau = -8.35 + 90 = 81.65^\circ$$

normal stress $\sigma_{avg} = \dfrac{\sigma_x+\sigma_y}{2} = \dfrac{100+70}{2} = 85\ N/mm^2$

source: diginotes.in

July 08/12 Marks

P04] The state of stress in two dimensionally stressed body is as shown. Determine the principal stresses, maximum shear stress and their planes.



Soln/.

Given $\sigma_x = -120\ N/mm^2$, $\sigma_y = -80\ N/mm^2$ and $\tau_{xy} = -60\ N/mm^2$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-120-80}{2} \pm \sqrt{\left(\frac{-120-(-80)}{2}\right)^2 + (-60)^2}$$

$\sigma_1 = -36.75\ N/mm^2$ and $\sigma_2 = -163.25\ N/mm^2$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-120-(-80)}{2}\right)^2 + (-60)^2}$$

$\tau_{max} = 63.25\ N/mm^2$

Orientations of Principal plane

$$\theta_{P_1} = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right] = \frac{1}{2}\tan^{-1}\left[\frac{2\times(-60)}{-120-(-80)}\right]$$

$\therefore \theta_{P_1} = 35.78$ and $\theta_{P_2} = \theta_{P_1} + 90° = 35.78 + 90$

$\therefore \theta_{P_2} = 125.78°$

Orientation of maximum shear stress plane

$$\theta_{S_1} = \frac{1}{2}\tan^{-1}\left[\frac{\sigma_y-\sigma_x}{2\tau_{xy}}\right] = \frac{1}{2}\tan^{-1}\left[\frac{-80-(-120)}{2\times(-60)}\right]$$

$\therefore \theta_{S_1} = -9.22°$

and

$\theta_{S_2} = -9.22 + 90 = 80.78°$

alternatively

$\theta_{S_1} = \theta_{P_1} + 45$

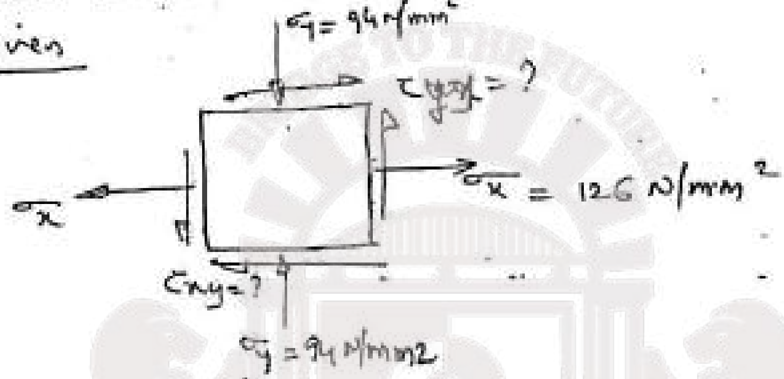
$= 35.78$

source: diginotes.in

Jan09-10marks(old scheme)

P35) At a point in a stressed material direct stresses 126 N/mm² tensile and 94 N/mm² compression are applied on planes at right angles to each other. If the maximum principal stress is limited to 146 N/mm², determine the shear stresses that may be allowed at the point in the same plane. Also determine the maximum shear stress

soln Given



maximum principal stress $\sigma_1 = 146\,N/mm^2$

$\sigma_x = 126\,N/mm^2$

$\sigma_y = -94\,N/mm^2$

$\tau_{xy} = ?$

$\tau_{max} = ?$

We have maximum principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$146 = \frac{126 - 94}{2} + \sqrt{\left(\frac{126-(-94)}{2}\right)^2 + \tau_{xy}^2}$$

$$146 = 16 + \sqrt{(110)^2 + \tau_{xy}^2}$$

or $$\sqrt{110^2 + \tau_{xy}^2} = 146 - 16 = 130$$

$$110^2 + \tau_{xy}^2 = (130)^2$$

$$\therefore \tau_{xy} = 69.28\,N/mm^2$$

source: diginotes.in

Minimum principal stress

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{126 - 94}{2} - \sqrt{\left(\frac{126-(-94)}{2}\right)^2 + 69.28^2}$$

$$\therefore \sigma_2 = 16 - 130 = -114 \ N/mm^2$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \frac{146 - (-114)}{2} = 130 \ N/mm^2$$

Point subjected to Biaxial Normal Stresses:

* Consider a point subjected to two mutually perpendicular stresses σ_x and σ_y as shown in figure(a).
* A portion of the element separated along plane BC whose normal is at an angle θ with respect to x-axis as shown in figure(b).
* The separated portion of the element is subjected
  i) induced normal stress σ_n and shear stress τ_{nt} on the plane BC.
  ii) Applied normal stresses σ_x on the plane AB and σ_y on the plane AC.

source: diginotes.in

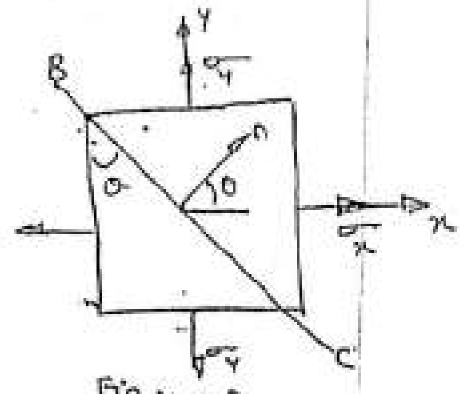


Figure (a)

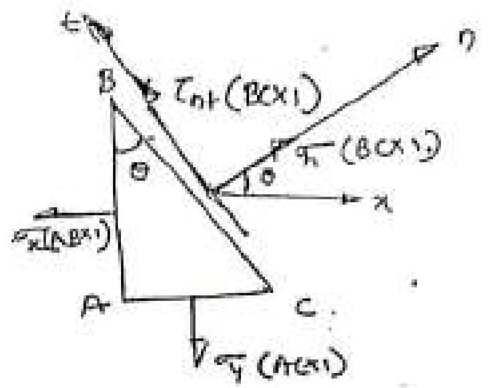


Figure (b)

Figure (c)

Stresses acting on a given plane

* To find the stresses acting on a given plane (plane BC) whose normal is at an angle θ with respect to x-axis, consider the equilibrium of the element ABC, shown in figure (b).
* Forces due to stresses σ_x and σ_y are resolved along n and t directions as shown in figure (c).

$$\Sigma F_n = 0$$

$$\sigma_n (BC\times1) - \sigma_x AB\cos\theta - \sigma_y AC\sin\theta = 0$$

$$\sigma_n = \sigma_x \frac{AB}{BC}\cos\theta + \sigma_y \frac{AC}{BC}\times\sin\theta$$

From figure (b) $\frac{AB}{BC} = \cos\theta$ and $\frac{AC}{BC} = \sin\theta$

$$\therefore \sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$$

Substituting $\cos^2\theta = \frac{1+\cos2\theta}{2}$ and $\sin^2\theta = \frac{1-\cos2\theta}{2}$ into above equation we get

$$\sigma_n = \sigma_x\left[\frac{1+\cos2\theta}{2}\right] + \sigma_y\left[\frac{1-\cos2\theta}{2}\right]$$

source: diginotes.in

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$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta \quad \text{--- (1)}$$

$\Sigma F_t = 0$

$\tau_{nt}(BC\times 1) + \sigma_x AB\sin\theta - \sigma_y AC\cos\theta = 0$

$\tau_{nt} = -\sigma_x \frac{AB}{BC}\sin\theta + \sigma_y \frac{AC}{BC}\cos\theta$

or $\tau_{nt} = -\sigma_x \cos\theta\sin\theta + \sigma_y \sin\theta\cos\theta$

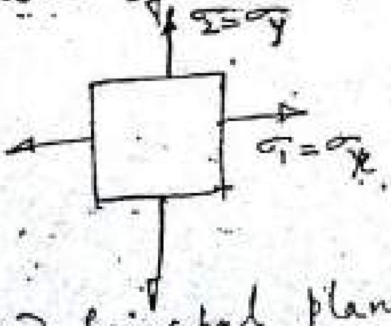
But $\sin\theta\cos\theta = \frac{\sin 2\theta}{2}$

$\therefore \tau_{nt} = -\sigma_x \frac{\sin 2\theta}{2} + \sigma_y \frac{\sin 2\theta}{2}$

$$\tau_{nt} = \sin 2\theta\left[\frac{\sigma_y - \sigma_x}{2}\right] \quad \text{--- (2)}$$

substituting the value of θ the inclination of normal to a given plane with respect to x-axis into equation (1) and (2) we get magnitudes of normal stress and shear stress acting on the plane.

Principal stresses :- The planes subjected to normal stresses σ_x and σ_y are the principal planes, since they are not subjected to any shear stress. Therefore the stresses σ_x and σ_y are the principal stresses. The orientations of these planes are given by $\theta_{P_{1,2}} = 0°$ and $90°$.



(a) Principal planes.

source: diginotes.in

Mechanics of Materials

Maximum and Minimum shear stresses

* shear stress on an arbitrary plane is given by

$$\tau_{n} = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta$$

The magnitude shear stress will be maximum when $\sin 2\theta = 1$ or $\theta = 45°$

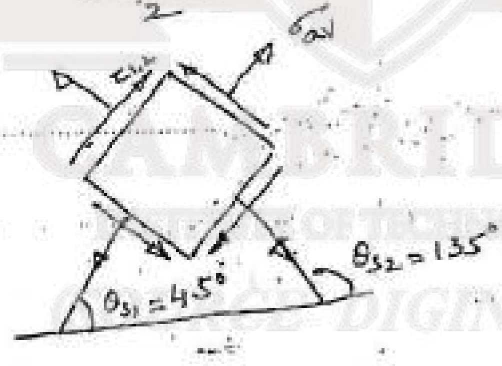
The minimum when $\sin 2\theta = -1$ or $\theta = 135°$.

* substituting these values of θ into equation ② we get the magnitudes of maximum and minimum shear stresses

$$\tau_{1,2} = \pm \frac{\sigma_y - \sigma_x}{2} = \pm \frac{\sigma_1 - \sigma_2}{2}$$

* Normal stress on the plane of maximum and minimum shear stresses is given by

$$\sigma_{av} = \frac{\sigma_1 + \sigma_2}{2}$$



Pb 6) A point in a machine member is subjected to principal stress (or 2 mutually perpendicular normal stress) of magnitude 30 MPa in tension and 100 MPa in compression. Determine
i) stresses acting on an element whose normal to one of its faces is oriented at an angle of 120° with

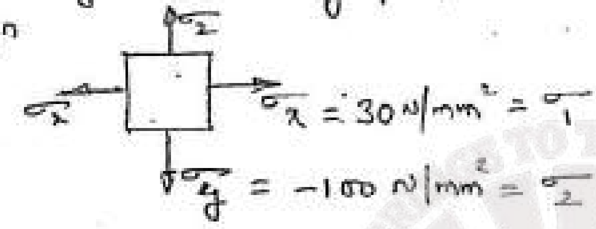
reference to x-axis

ii) Maximum and minimum shear stress and their orientations.

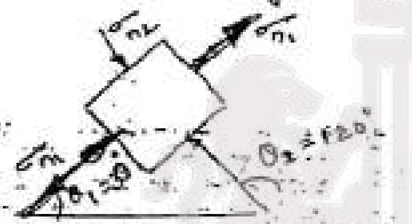
iii) Normal stresses acting on maximum and minimum shear stress planes.

iv) Prove that the sum of normal stresses at a point on any 2 mutually perpendicular planes is constant

Soln



(i) Stress acting on the element:-



When the orientation of normal to one of the elements faces is $\theta_2 = 120°$, the orientation of normal to the other faces will be $\theta = 120 - 90 = 30°$

Now,

normal stress, $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$

Substitute $\theta_1 = 30°$, we get

$$\sigma_{n1} = \frac{30 - 100}{2} + \frac{30 - (-100)}{2} \cos(2 \times 30) + 0$$

$$\therefore \sigma_{n1} = -35 + 65 \times \cos 60 = -2.5 \text{ MPa}$$

Now, Substitute $\theta_2 = 30 + 90 = 120°$

$$\sigma_{n2} = \frac{30 - 100}{2} + \frac{30 - (-100)}{2} \cos 240 + 0$$

$$\sigma_{n2} = -35 + 65 \times \cos 240 = -67.5 \text{ MPa}$$

Shear stress $\tau_t = \frac{\sigma_y - \sigma_x}{2} \times \sin 2\theta + \tau_{xy} \cos 2\theta$

Substitute $\theta = 30°$, we get, $\tau_{t} = \frac{-100 - 30}{2} \sin(2 \times 30) + 0$

source: diginotes.in

$$\tau_{nt} = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta + \tau_{xy} \sin 2\theta$$

substitute $\theta = 30°$, we get

$$\tau_{nt} = \frac{-100-30}{2} \times \sin(2\times 30) + 0 = -56.29 \text{ N/mm}^2$$

ii) Maximum and minimum shear stress

$$\tau_{1,2} = \pm \frac{\sigma_1 - \sigma_2}{2} = \pm \frac{30-(-100)}{2}$$

$$= \pm 65 \text{ N/mm}^2$$

∴ maximum shear stress $\tau_1 = 65 \text{ N/mm}^2$

minimum shear stress $\tau_2 = -65 \text{ N/mm}^2$

The orientations of maximum and minimum shear stress plane are $\theta_{s_1} = 45°$ and $\theta_{s_2} = \theta_{s_1} + 90 = 45 + 90$
$= 135°$

iii) Normal stresses acting on maximum and minimum shear stress planes

$$\sigma_{avg} = \frac{\sigma_1 + \sigma_2}{2} = \frac{30+(-100)}{2} = -35 \text{ MPa}$$

iv) T.P.T $\sigma_{n_1} + \sigma_{n_2} = \sigma_1 + \sigma_2$

Now, $\sigma_{n_1} + \sigma_{n_2} = -2.5 + (-67.5) = -70 \text{ N/mm}^2$

$$\sigma_1 + \sigma_2 = (30-100) = -70 \text{ N/mm}^2$$

Hence $\sigma_{n_1} + \sigma_{n_2} = \sigma_1 + \sigma_2$ (proved)

source: diginotes.in

$$\boxed{\sigma_{1,2} = \pm \tau_{xy}}$$

Maximum and minimum shear stress:-

* shear stress on an arbitrary plane is given by

$$\tau_{nt} = \tau_{xy} \cdot \cos 2\theta$$

shear stress will be maximum when $\cos 2\theta = 1$

i.e. $\theta_{s_1} = 0$

shear stress will be minimum when $\cos 2\theta = -1$

i.e., $\theta_{s_2} = 90°$

substituting $\theta_{s_1} = 0$, we get

$\therefore \tau_1 = \tau_{xy} \cos(2 \times 0)$

$\tau_1 = \tau_{xy}$

substituting $\theta_{s_2} = 90°$ we get

$\therefore \tau_2 = \tau_{xy} \cos(2 \times 90°)$

$\therefore \tau_2 = -\tau_{xy}$

$$\tau_{1,2} = \pm \tau_{xy}$$

* Magnitudes of principal stresses and maximum and minimum shear stress are all numerically equal to the shear stress τ_{xy}.
* The magnitudes of normal stresses acting on the planes of maximum and minimum shear stress is zero

$$\sigma_{avg} = \frac{\sigma_1 + \sigma_2}{2}$$

$$= \frac{\tau_{xy} - \tau_{xy}}{2}$$

$$\therefore \sigma_{avg} = 0$$

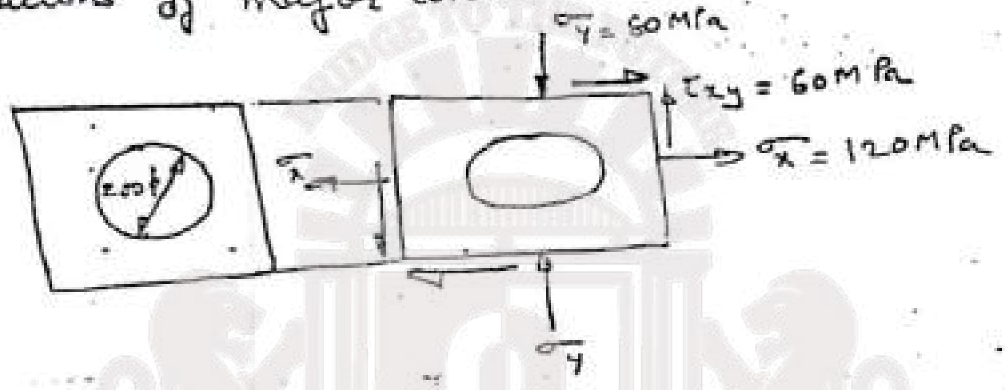
source: diginotes.in

7) A square plate is subjected to a tensile stress of 120 MPa and compressive stress 80 MPa on its 2 perpendicular faces. In addition a shear stress of magnitude 60 MPa is also applied. A circle of 200 mm diameter drawn on it is converted to an ellipse. Take E = 200 GPa and γ = 0.3. Determine :-

i) Dimensions of major and minor axes of the ellipse.

ii) Orientations of major and minor axes.

soln:



Young's modulus $E = 200\ GPa = 2 \times 10^5\ N/mm^2$

Poisson's ratio $\frac{1}{m} = \gamma = 0.3$

Major axis (x-drn) = diameter of the circle + change in length in x drn,

$= d + dL_1$

Now strain in x drn, $\epsilon_1 = \frac{dL_1}{d_0}$ or $dL_1 = \epsilon_1 d_0$

∴ Major axis $= d + dL_1$

$= d + \epsilon_1 d_0$

and Minor axis = diameter of the circle + change in length in Y drn

$= d + \epsilon_2 d$

where strain in Y direction $\epsilon_2 = \frac{dL_2}{d}$

The circle is converted to ellipse due to the 2 mutually perpendicular principal strains acting along the principal stress directions.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{120-80}{2} \pm \sqrt{\left[\frac{120-(-80)}{2}\right]^2 + 60^2} = 20 \pm \sqrt{100^2 + 60^2}$$

$$= 20 \pm 116.62$$

$\therefore \sigma_1 = 136.62\ N/mm^2$ and $\sigma_2 = -96.62\ N/mm^2$

Principal strain along direction 1-1

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{1}{m}\frac{\sigma_2}{E}$$

$$= \frac{136.62}{2\times10^5} - \frac{0.3\times(-96.92)}{2\times10^5}$$

$$\therefore \epsilon_1 = 8.28\times10^{-4}$$

Principal strain along direction 2-2

$$\epsilon_2 = \frac{\sigma_2}{E} - \frac{\sigma_1}{mE}$$

$$= \frac{-96.62}{2\times10^5} - \frac{136.62\times0.3}{2\times10^5}$$

$$\therefore \epsilon_2 = -0.688\times10^{-3}$$

$\therefore$ Length of major axis $= d + \epsilon_1 d = 200 + 8.28\times10^{-4}\times200 = 200.17$

$\therefore$ Length of minor axis $= d + \epsilon_2 d = 200 + (-0.688\times10^{-3})\times200 = 199.86\ mm$

source: diginotes.in

ii) Orientation of the axes

$$\theta_{P1} = \frac{1}{2} \tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right]$$

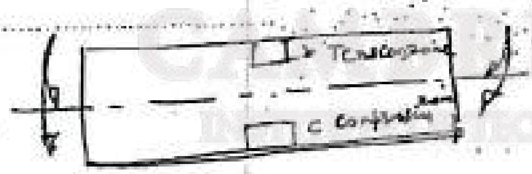
$$= \frac{1}{2} \tan^{-1}\left[\frac{2 \times 160}{120-(-80)}\right]$$

$$\therefore \theta_{P1} = 15.5^\circ$$

and $\theta_{P2} = \theta_{P1} + 90$

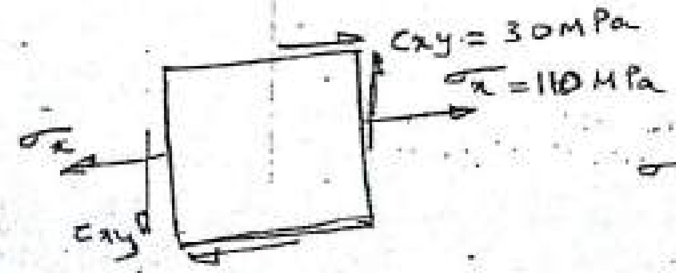
$$\therefore \theta_{P2} = 15.5 + 90 = 105.5^\circ$$

Pb 8] A point in a beam is subjected to maximum tensile stress of 110 MPa and shear stress of 30 MPa. Find the magnitudes and directions of principal stresses. If the point in the beam is in the compression zone under the same magnitude of ~~bending~~ zone under the same magnitude of bending bending stress and shear stress, find the magnitudes of principal stresses and their directions



Solution

(i) Element in Tension zone



$\sigma_{1,2} = ?$ θ_{P1} & $\theta_{P2} = ?$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{110+0}{2} \pm \sqrt{\left(\frac{110-0}{2}\right)^2 + 30^2}$$

$\therefore \quad \sigma_1 = 117.7\,MPa$ and $\sigma_2 = -7.7\,MPa$

$$\theta_{P_1} = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right]$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{2\times 30}{110-0}\right]$$

$\therefore \theta_{P_1} = 14.3^\circ$

and $\theta_{P_2} = 14.3 + 90 = 104.3^\circ$

Element in Compression zone

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-110}{2} \pm \sqrt{\left(-\frac{110}{2}\right)^2 + 30^2} = -55 + 62.65$$

$\therefore \sigma_1 = 7.65\,N/mm^2$ and $\sigma_2 = -117.65\,N/mm^2$

$$\theta_{P_1} = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right] = \frac{1}{2}\tan^{-1}\left[\frac{2\times 30}{-110-0}\right]$$

$\therefore \theta_{P_1} = -14.3^\circ$

$\theta_{P_2} = -14.3 + 90 = 75.69^\circ$

source: diginotes.in

Mohr's circle method:-

* Mohr's circle is used to get an immediate picture of stresses induced on various planes. Normal stresses and shear stress acting on an arbitrary plane passing through a point can be represented as points on a circle.

* The circle representation of the stress system can be developed by equations for normal stress and shear stress on an arbitrary plane.

* Normal and shear stresses on an arbitrary plane are given by,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \quad \text{——} \ (1)$$

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \quad \text{——} \ (2)$$

Equation (1) can be rearranged as

$$\sigma_n - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \quad \text{——} \ (3)$$

Squaring and adding equation (2) and (3) we get

$$\left[\sigma_n - \frac{\sigma_x + \sigma_y}{2}\right]^2 + \tau_{nt}^2 = \left[\frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta\right]^2 + \left[-\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta\right]^2$$

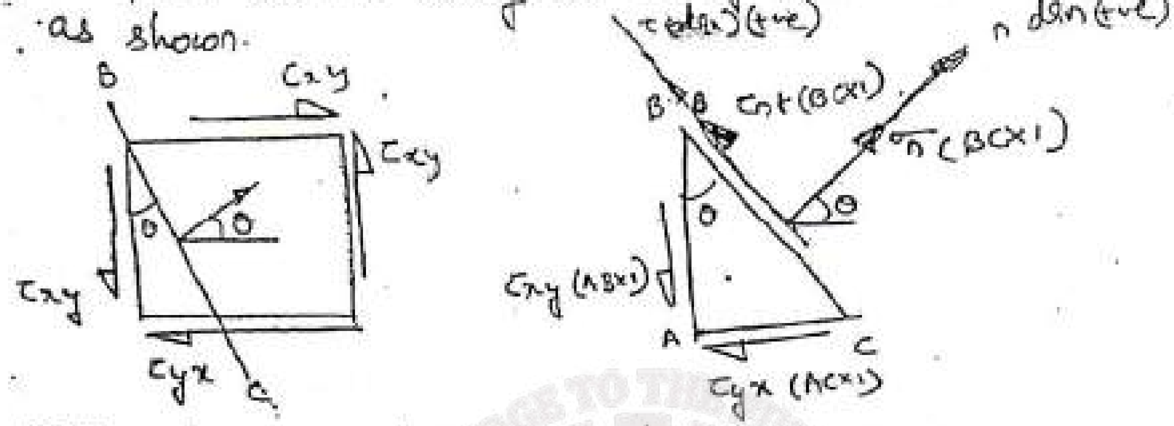
Simplifying

$$\left[\sigma_n - \frac{\sigma_x + \sigma_y}{2}\right]^2 + \tau_{nt}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \quad \text{——} \ (4)$$

source: diginotes.in

Point subjected to pure shear stress

* A point in a body is subjected to pure shear stress as shown.



* The forces can be resolved as:-

* **Stresses on a given plane**

For equilibrium of forces in n direction,

$\Sigma F_n = 0$

$\sigma_n \times (BC \times 1) - \tau_{xy} AB \sin\theta - \tau_{yx} AC \cos\theta = 0$

$\sigma_n = \tau_{xy} \frac{AB}{BC} \sin\theta + \tau_{yx} \frac{AC}{BC} \cos\theta$

Now in △le ABC

$\frac{AB}{BC} = \cos\theta$ and $\frac{AC}{BC} = \sin\theta$ and $\tau_{xy} = \tau_{xy}$

$\sigma_n = \tau_{xy} \cos\theta \sin\theta + \tau_{xy} \sin\theta \cos\theta$

We have $\sin\theta \cos\theta = \frac{\sin 2\theta}{2}$

$\therefore \sigma_n = \tau_{xy} \frac{\sin 2\theta}{2} + \tau_{xy} \frac{\sin 2\theta}{2} = 2\tau_{xy} \frac{\sin 2\theta}{2}$

$$\therefore \boxed{\sigma_n = \tau_{xy} \sin 2\theta}$$

source: diginotes.in

For equilibrium in t - direction,

$\Sigma F_t = 0$

$\tau_{nt}(BC \times 1) - \tau_{xy} AB \cos\theta - \tau_{yx} AC \sin\theta = 0$

$\tau_{nt} = \tau_{xy} \frac{AB}{BC} \cos\theta - \tau_{yx} \frac{AC}{BC} \sin\theta$

$\therefore \tau_{nt} = \tau_{xy} \cos^2\theta - \tau_{yx} \sin^2\theta$

Substituting $\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ & $\tau_{xy} = \tau_{yx}$

$\therefore \tau_{nt} = \tau_{xy}\left[\frac{1+\cos 2\theta}{2}\right] - \tau_{xy}\left[\frac{1-\cos 2\theta}{2}\right]$

$= \frac{2\tau_{xy}\cos 2\theta}{2}$

$$\boxed{\tau_{nt} = \tau_{xy} \cos 2\theta}$$

Principal stresses

Normal stress acting on the arbitrary plane is given by

$\sigma_n = \tau_{xy} \sin 2\theta$

The magnitude of normal stress is maximum,

when $\sin 2\theta = 1$ i.e., at $\theta_{P1} = 45^\circ$ and

The magnitude of normal stress is minimum

when $\sin 2\theta = -1$ i.e., at $\theta_{P2} = 135^\circ$.

Substituting the above values in σ_n equation we get principal stress and minimum principal stress,

At $\theta_{P1} = 45^\circ$	At $\theta_{P2} = 135^\circ$
$\sigma_1 = \tau_{xy}$	$\sigma_2 = -\tau_{xy}$

source: diginotes.in

3) The distances OE and OA measured on σ axis are major principal stress and minor principal stress respectively.
The angle of Mohr's circle are twice the angle measured from the planes. Therefore θ_{P1} & θ_{P2} gives the orientation of major and minor principal stresses w.r.t x-axis.

4) The line CF representing normal to maximum shear stress is at an angle $2\theta_s$ measured in anti-clockwise direction with respect to x-axis line CX).
The point F(σ_{ave}, τ_1) gives the values of maximum shear stress τ_1 and normal stress σ_{avg} on the plane of maximum shear stress.

τ_1 = CF (radius of the circle

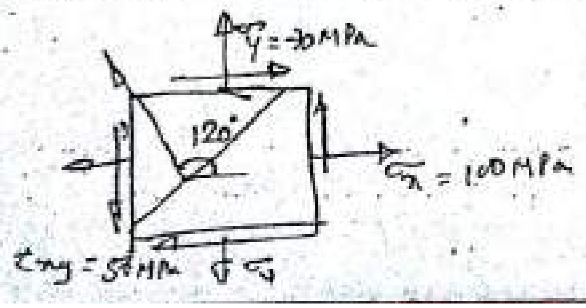
OC = σ_{avg}

5) The plane whose normal is at an angle θ with respect to x-axis is obtained by drawing the line CP at an angle 2θ in counter clockwise direction with respect to x-axis (line CX) on the Mohr's circle

Now OP' = σ_n and

PP' = τ_{nt}

Pb9) An element is subjected to the given stress as shown. Solve the problem using Mohr's circle method



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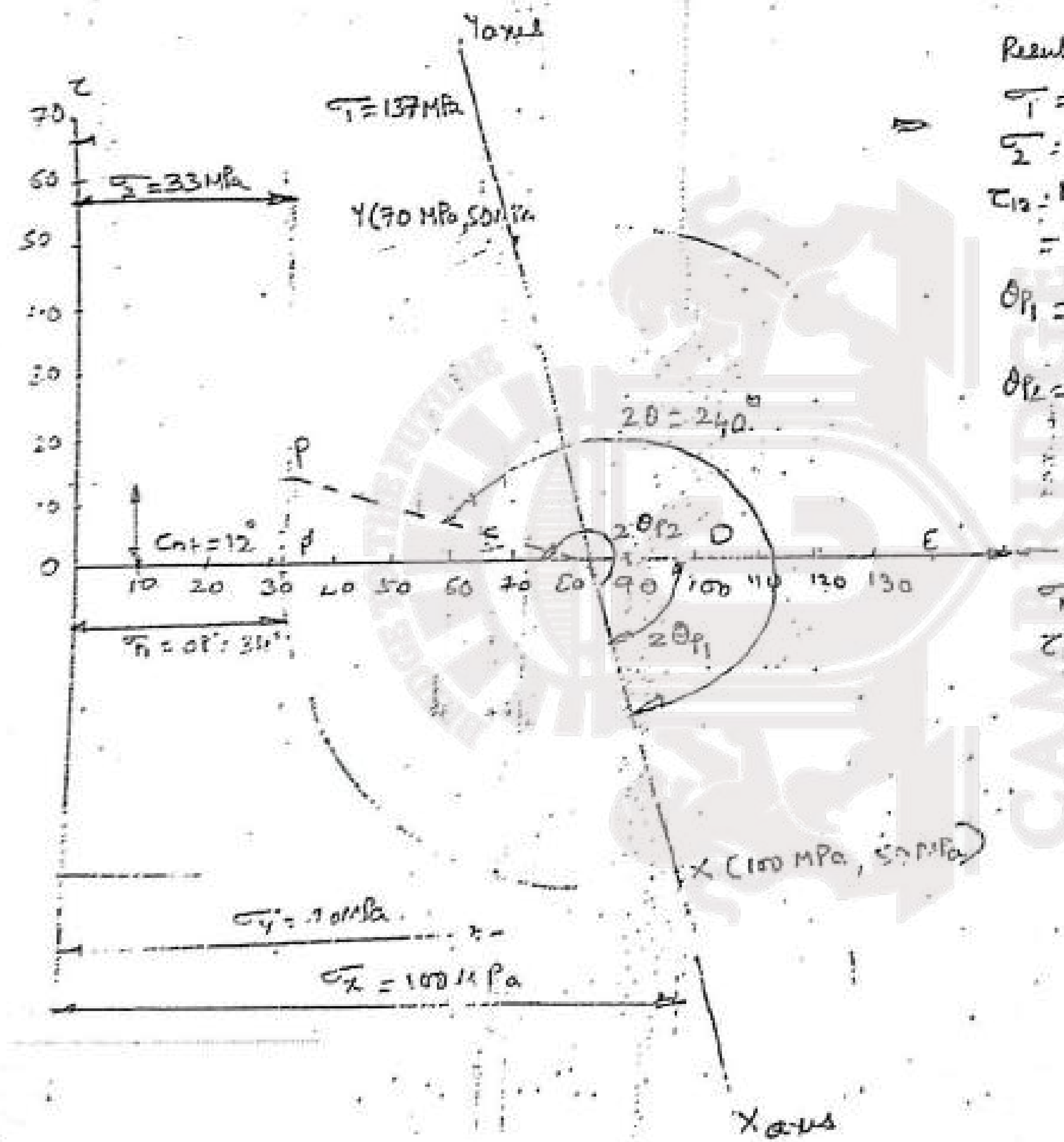


Results

$\sigma_1 = OE = 137$ MPa

$\sigma_2 = OA = 33$ MPa

τ_{12} = Max & Min shear stress

= R = ± 52 MPa

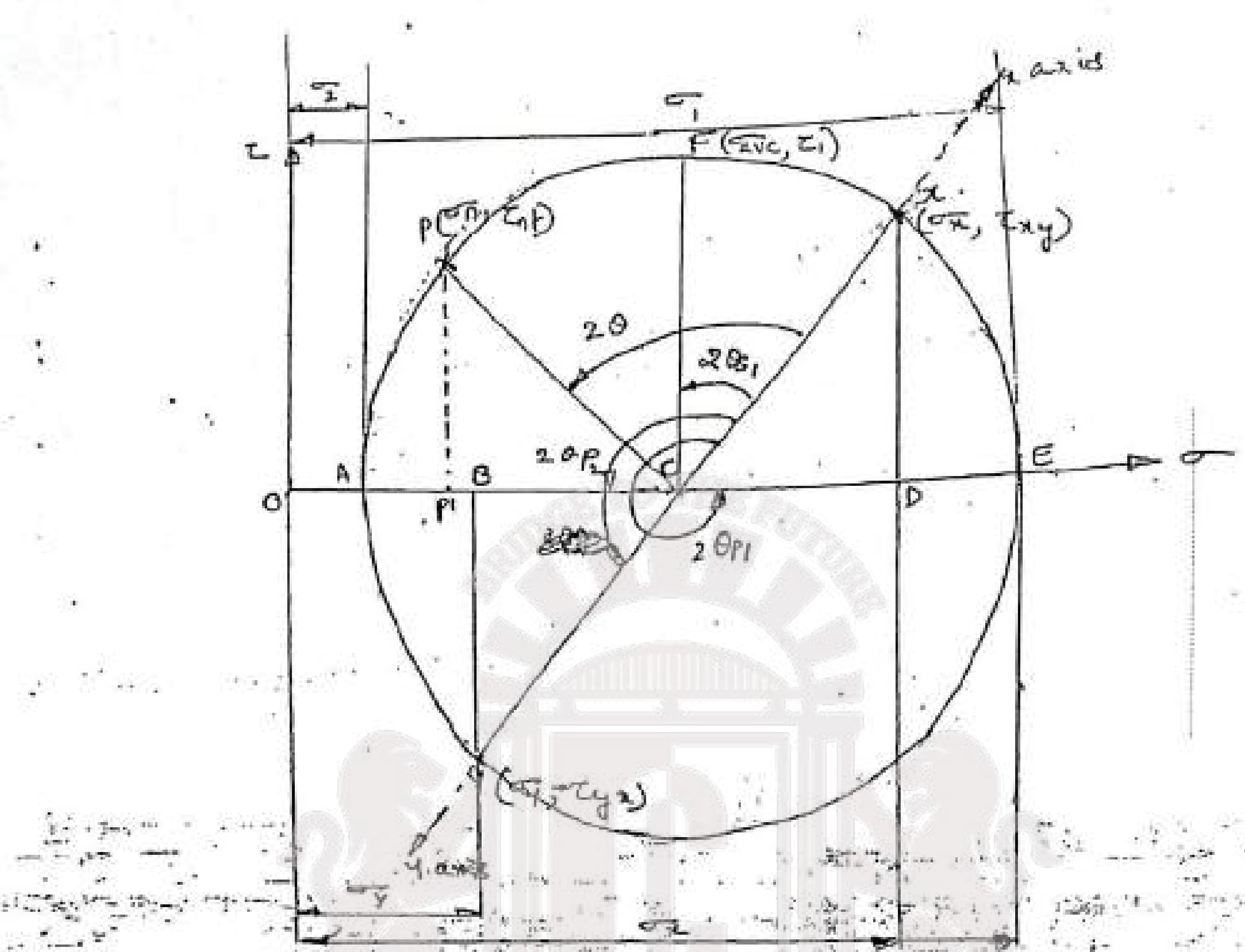
$\theta_{P1} = \frac{72}{2} = 36°$

$\theta_{P2} = \frac{180+72}{2} = 126$

$\sigma_n = OF' = 34$

$\tau_{nt} = FF' = 12°$

source: diginotes.in



The signs followed for shear stress in case of Mohr's circle is different from the followed in case of analytical method

Negative shear on x-face & positive shear on y-face

Positive shear on x-face and negative shear on y-face

Join the points x and y by a straight line XY, which is the diameter of the circle with its centre C on the σ axis. Draw the circle with C as centre and CX as radius

The line CX and CY represent the x and y axes.

source: diginotes.in

given stress system, the stresses σ_x, σ_y and are constants. Whereas σ_n and τ_{nt} are the variables which depend upon the orientation of the plane. ∴ we can write

$$\left(\frac{\sigma_x + \sigma_y}{2}\right) = c \quad \text{and} \quad \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 = R^2$$

where c and R are constants. Substituting these values in equation (4) we get,

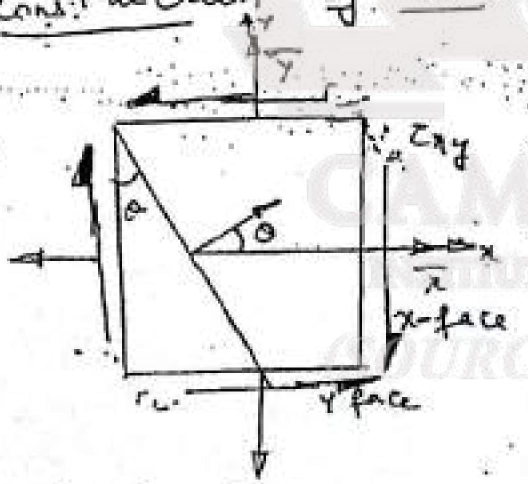
$$(\sigma_n - c)^2 + \tau_{nt}^2 = R^2$$

The above equation represents a circle with radius,

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$ and with position of its

centre $c = \dfrac{\sigma_x + \sigma_y}{2}$

Construction of Mohr's circle



Consider an element in a body subjected to stress system as shown in the adjoining figure. The steps followed for drawing the Mohr's circle are given below

1) Mark the stresses (σ_x, τ_{xy}) acting on x face. and (σ_y, τ_{yx}) acting on the y face by the points x and y.

source: diginotes.in

THIN AND THICK CYLINDERS

Stresses in thin cylinders, changes in dimensions of cylinder (diameter, length and volume), Thick cylinders subjected to internal and external pressures (Lame's equation). [6 Hours]
[Compound cylinders not included]

Introduction :-

* Closed containers known as pressure vessels are used to store liquids, gases and compressed air etc. Typical examples of pressure vessels are steam engine cylinders, water tanks, compressed air/gas storage tanks and steam boilers etc, which store fluid or gas at high pressure
* condition
* The shapes of pressure vessels generally used are cylinder and sphere

Thin and Thick Vessels :-

* The pressure vessels are classified into two groups, thin vessels and thick vessels based on the ratio of wall radius R to wall thickness t.

Thin Vessel $\frac{R}{t} > 10$

Thick Vessel $\frac{R}{t} \leq 10$

Stresses in thin walled cylinders

* Consider a cylinderical vessel of inner radius 'r' and wall thickness containing a fluid under pressure.

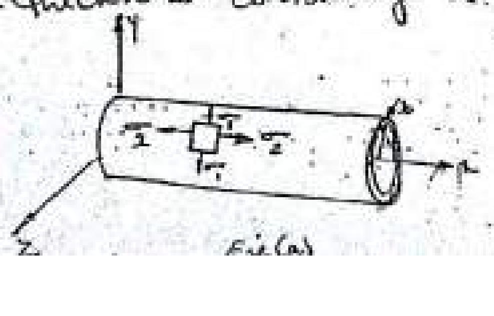


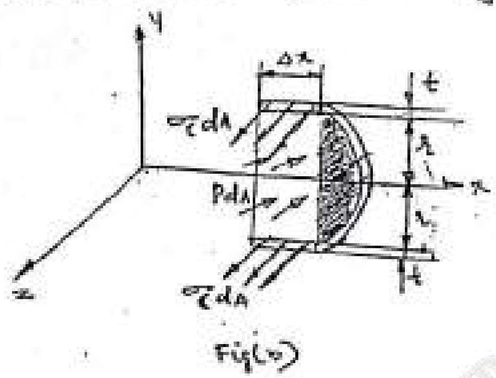
Fig (a)

* The stresses σ_1 is known as the hoop's stress (or circumferential stress or tangential stress) and

source: diginotes.in

The stress σ_z is called longitudinal stress.

<u>Hoops stress or circumferential stress</u>



Fig(a)

→ A portion of the cylinder and its contents bounded by xy plane and of thickness Δx is selected as shown.

→ Bursting forces acting normal to the longitudinal plane induces normal stress on the longitudinal sections of the wall as shown. This stress acting tangentially w.r.t circumference of the cylinder is known as hoop stress or circumferential stress.

∴ force acting on the circumference $dF_c = \sigma_c \, dA$

$= \sigma_c \, \Delta x \times 2t$.

∴ force acting on entire length is given by $F_c = \sigma_c 2t \int_0^L \Delta x$

$= 2\sigma_c t L$ —①

⇒ The bursting forces opposing this forces can be written as, $dF_b = PdA = P\,2r\,\Delta x$

∴ force acting on entire length is given by $F_b = 2Pr \int_0^L \Delta x$

$= 2PrL$. —①

For equilibrium $\Sigma F_z = 0$

$2\sigma_c t L - P(2r)L = 0$

∴ or $\boxed{\sigma_c = \frac{Pr}{t}}$ or $\boxed{\sigma_c = \frac{PD}{2t}}$ —①

where D is the internal diameter of the cylinder.

CAMBRIDGE INSTITUTE OF TECHNOLOGY (SOURCE DIGINOTES)

source: diginotes.in

Longitudnal stress σ_L :-

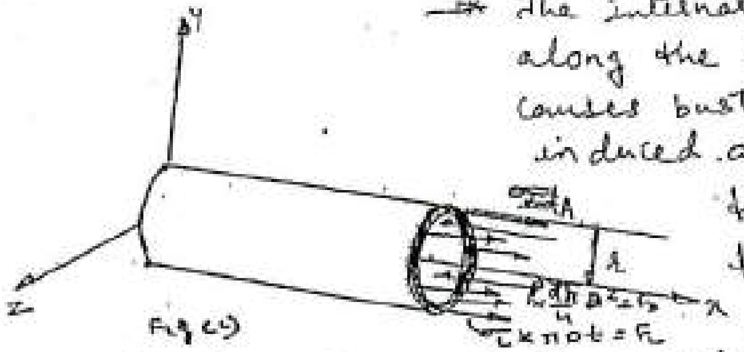


Fig (c)

→ The internal pressure acting along the longitudnal direction causes busting forces to be induced along the same direction that is normal to the transverse plane (i.e. z plane)

→ The bursting force may be calculated as

F_b = internal pressure × cross sectional area of the vessel circle

$\therefore F_b = P \times \frac{\pi}{4} D^2$

→ The resistance exerted by the vessel is the

F_L = longitudnal stress × c/s area of the vessel

$= \sigma_L \times \pi D t$

→ For Equilibrium, $\Sigma F_x = 0$

$$\sigma_L \pi D t - P \frac{\pi}{4} D^2 = 0$$

or $\sigma_L \pi D t = \frac{P \pi}{4} D^2$

or $$\boxed{\sigma_L = \frac{PD}{4t}} \quad \text{—— (2)}$$

→ From equations (1) and (2), we find that

$$\sigma_C = \frac{\sigma_L}{2}$$

* A small element on the surface of the pressure vessel is subjected to circumferential stress σ_C and longitudnal stress σ_L as shown in fig (a). The element is subjected to maximum shear stress on a plane which is at an angle of 45° w.r.t the ~~log~~ longitudnal axis

∴ Maximum shear stress, $\tau_{max} = \frac{\sigma_C - \sigma_L}{}$

CAMBRIDGE (SOURCE DIGINOTES)

Change in dimensions :-

* Based on Hook's law, we can find the circumferential and longitudinal strains induced. Diameter, length and hence volume will increase when the cylindrical pressure vessel is subjected to internal pressure p.

* Circumferential strain is given by

$$\epsilon_C = \frac{\sigma_C}{E} - \frac{\sigma_L}{mE} = \frac{PD}{2tE} - \frac{PD}{4tE} \times \frac{1}{m}$$

$$\therefore \epsilon_C = \frac{PD}{2tE}\left[1 - \frac{1}{2m}\right].$$

Change in diameter, $\delta D = \epsilon_C D$

* Longitudinal strain is given by,

$$\epsilon_L = \frac{\sigma_L}{E} - \frac{\sigma_C}{mE}$$

$$= \frac{PD}{4tE} - \frac{PD}{2tE} \times \frac{1}{m} \times \frac{2}{2}$$

$$\therefore \epsilon_L = \frac{PD}{4tE}\left[1 - \frac{2}{m}\right]$$

Change in length $\delta L = \epsilon_L \times L$

* Volume of the cylinder is given by, $V = \frac{\pi}{4} D^2 L$

Change in volume is given by

$$\delta V = \frac{\pi}{4} D^2 (\delta L) + \frac{\pi}{4} L (2D\,\delta D)$$

Dividing the equation throughout by $V = \frac{\pi}{4} D^2 L$, we get

$$\frac{\delta V}{V} = \frac{\frac{\pi}{4} D^2 \delta L}{\frac{\pi}{4} D^2 L} + \frac{\frac{\pi}{4} L\, 2D\,\delta D}{\frac{\pi}{4} D^2 L} = \frac{\delta L}{L} + \frac{2\,\delta D}{D}$$

But $\frac{\delta V}{V}$ is volumetric strain ϵ_v

$$\therefore \epsilon_v = \frac{\delta V}{V} = \epsilon_L + 2\epsilon_C$$

source: diginotes.in

Change in volume $dV = \epsilon_v V = (2\epsilon_c + \epsilon_L)V$

$$\therefore \epsilon_v = \left\{\frac{2PD}{2tE}\left[1-\frac{1}{2m}\right] + \frac{PD}{4tE}\left[1-\frac{2}{m}\right]\right\}V$$

$$= \left\{\frac{PD}{tE} - \frac{1}{2m}\frac{PD}{tE} + \frac{PD}{4tE} - \frac{2}{m}\times\frac{PD}{4tE}\right\}V$$

$$= \frac{PD}{4tE}\left[4 - \frac{2}{m} + 1 - \frac{2}{m}\right]V$$

$$= \frac{PD}{4tE}\left[5 - \frac{4}{m}\right]V$$

Pb] A thin cylinderical shell 0.6 diameter and 0.9 m long is subjected to internal stresses pressure 1.2 N/mm². Thickness of cylinder of wall is 15 mm. Determine
i) Longitudinal stress circumferential stress and maximum shear stress induced ii) Change in diameter, length and volume
Take E = 200 GPa and $\frac{1}{m} = 0.3$

Soln
Given

diameter of cylinder $D = 0.6m = 600mm$
length " " , $L = 0.9m = 900mm$
Wall thickness of cylinder, $t = 15$ mm
internal pressure $P = 1.2$ N/mm²
Young's modulus $E = 200$ GPa $= 2\times10^5$ N/mm²
Poisson's ratio $\frac{1}{m} = 0.3$

$\sigma_c = ?$ $\sigma_L = ?$ $\tau_{max} = ?$ $\delta L = ?$ $\delta V = ?$ $\delta d = ?$

i). Stresses induced

Longitudnal stress $\sigma_L = \frac{PD}{4t} = \frac{1.2\times600}{4\times15} = 12$ N/mm²

Circumferential stress $\sigma_c = \frac{PD}{2t} = \frac{1.2\times600}{2\times15} = 24$ N/mm²

Maximum shear stress $\tau = \frac{\sigma_c - \sigma_L}{2} = \frac{24-12}{2} = 6$ N/mm²

source: diginotes.in

ii) Change in dimensions

Longitudinal strain $\epsilon_L = \frac{\sigma_L}{E} - \frac{\sigma_c}{mE} = \frac{12}{2\times10^5} - \frac{24}{2\times10^5}\times0.3 = 2.4\times10^{-5}$

change in length $\delta L = \epsilon_L \times L = 2.4\times10^{-5}\times900 = 0.0216$ mm

Circumferential strain $\epsilon_c = \frac{\sigma_c}{E} - \frac{\sigma_L}{mE} = \frac{24-(0.3\times12)}{2\times10^5} = 1.02\times10^{-4}$

Change in diameter $\delta D = \epsilon_c \times D = 1.02\times10^{-4}\times600 = 0.0612$ mm

Volumetric strain $\epsilon_v = 2\epsilon_c + \epsilon_L = 2\times1.02\times10^{-4} + 2.4\times10^{-5} = 2.28\times10^{-4}$

change in volume $\delta V = \epsilon_v \times V = 2.28\times10^{-4}\times\frac{\pi}{4}\times600^2\times900 = 58.02\times10^3\ mm^3$

Jan 09 (old scheme - 6 marks)

Pb 2) A cylinder thin drum 800 mm in diameter and 3 m long has a shell thickness of 10 mm. If the drum is subjected to an internal pressure of 2.5 N/mm². Calculate the change in diameter, change in length and change in volume. Take E = 200 GPa. Poisson's ratio = 0.25

Soln

Given

diameter of cylinder D = 800 mm	Internal pressure P = 2.5 N/mm²
length " " L = 3 m = 3000 mm	Young's modulus E = 200 GPa = 2×10^5 N/mm²
thickness t = 10 mm	Poisson's ratio $\frac{1}{m} = 0.25$

$\delta D = ?$ $\delta L = ?$ $\delta V = ?$

Longitudinal stress $\sigma_L = \frac{Pd}{2t} = \frac{2.5\times800}{2\times10} = 100$ N/mm²

Circumferential stress $\sigma_c = \frac{Pd}{4t} = \frac{2.5\times100}{4\times10} = 50$ N/mm²

Longitudinal strain $\epsilon_L = \frac{\sigma_L}{E} - \frac{\sigma_c}{mE} = \frac{100 - 50\times0.25}{2\times10^5} = 4.375\times10^{-4}$

source: diginotes.in

change in length $\delta L = \epsilon_L \times L = 4.375 \times 10^{-4} \times 3000 = 1.3125$ mm

Change in Circumferential strain $\epsilon_c = \frac{\sigma_c}{E} - \frac{\sigma_L}{mE} = \frac{50 - 0.25 \times 100}{2 \times 10^5} = 1.25 \times 10^{-4}$

change in diameter $\delta D = \epsilon_c D = 1.25 \times 10^{-4} \times 800 = 0.1$ mm.

Volumetric strain $\epsilon_v = 2\epsilon_c + \epsilon_L = 2 \times 1.25 \times 10^{-4} + 4.375 \times 10^{-4}$

$= 2.5 \times 10^{-4} = 6.875 \times 10^{-4}$

change in volume $\delta V = \epsilon_v \times V = 6.875 \times 10^{-4} \times \frac{\pi}{4} \times 800^2 \times 3000$

$= 1.0367 \times 10^6$ mm^3

July08 - 10 Marks

Pb 3] A thin cylinderical shell 1.2 m in diameter and 3 m long has a metal wall of thickness of 12 mm. It is subjected internal fluid pressure of 3.2 MPa. Find the circumferential and longitudinal stress in the wall. Determine change in length, diameter and volume of the cylinder. Assume E = 210 GPa and $\mu = 0.3$

Soln

Given

Diameter of cylinder $D = 1.2$ m $= 1200$ mm

Length of cylinder $L = 3$ m $= 3000$ mm

thickness $t = 12$ mm

Internal pressure $P = 3.2$ MPa $= 3.2$ N/mm^2

Young's modulus $E = 210$ GPa $= 2.1 \times 10^5$ N/mm^2

Poisson's ratio $\mu = \frac{1}{m} = 0.3$

$\delta L = ?$, $\delta D = ?$, $\delta V = ?$

Circumferential stress $\sigma_c = \frac{PD}{2t} = \frac{3.2 \times 1200}{2 \times 12} = 160$ N/mm^2

Longitudinal stress $\sigma_L = \frac{PD}{4t} = \frac{3.2 \times 1200}{4 \times 12} = 80$ N/mm^2

source: diginotes.in

Circumferential strain $\epsilon_c = \frac{\sigma_c}{E} - \mu\frac{\sigma_L}{E} = \frac{160 - 0.3\times 80}{2.1\times 10^5} = 6.48\times 10^{-4}$

change in diameter $\delta d = \epsilon_c \times d = 6.48\times 10^{-4} \times 1200 = 0.7776$ mm

Longitudinal strain $\epsilon_L = \frac{\sigma_L}{E} - \frac{\mu\sigma_c}{E} = \frac{80 - 0.3\times 160}{2.1\times 10^5} = 1.524\times 10^{-4}$

change in length $\delta L = \epsilon_L \times L = 1.524\times 10^{-4} \times 3000 = 0.4572$ mm

Volumetric strain $\epsilon_v = 2\epsilon_c + \epsilon_L = 2\times 6.48\times 10^{-4} + 1.524\times 10^{-4}$

$= 1.4484\times 10^{-3}$

change in volume $\delta V = \epsilon_v \times V = 1.4484\times 10^{-3} \times \frac{\pi}{4} \times 1200^2 \times 3000$

$= 4.914\times 10^{6}$ mm^3

PS4). A cylindrical pressure vessel of 1m inner diameter and 1.5m long is subjected to an internal pressure P. Thickness of the cylinder wall is 15mm. Taking allowable stress for cylinder material as 90 MPa. Determine

i) magnitude of maximum internal pressure P that the pressure vessel can withstand and

(ii) change in dimensions. Take E = 200 GPa and $\gamma = 0.3$

Soln

Given

Diameter of cylinder $D = 1.5m = 1000$ mm	Young's modulus $E = 200$ GPa
Length " $L = 1.5m = 1500$ mm	or $E = 2\times 10^5$ N/mm^2
thickness $t = 15$ mm.	Poisson's ratio $\frac{1}{m} = \gamma = 0.3$
Allowable stress for cylinder material $\sigma = 90$ MPa $= 90$ N/mm^2	$P = ?,\ \delta D, dL, dV = ?$

i) Maximum internal pressure :-

The magnitude of maximum pressure is found based on circumferential stress, since it is the maximum stress induced in the cylinder wall. i.e. $\sigma_c = 90$ N/mm^2

source: diginotes.in

Circumferential stress, $\sigma_c = \frac{PD}{2t}$

or $90 = \frac{P \times 1000}{2 \times 15}$ or $P = 2.7\ N/mm^2$

Longitudinal stress

$\sigma_L = \frac{\sigma_c}{2} = \frac{90}{2} = 45\ N/mm^2$

ii) Change in dimensions :-

Circumferential strain $\epsilon_c = \frac{\sigma_c}{E} - \frac{\sigma_L}{mE} = \frac{90 - 45 \times 0.3}{2 \times 10^5} = 3.825 \times 10^{-4}$

Change in diameter $\delta D = \epsilon_c D = 3.825 \times 10^{-4} \times 1000 = 0.3825\ mm$

Longitudinal strain $\epsilon_L = \frac{\sigma_L}{E} - \frac{\sigma_c}{mE} = \frac{45 - 90 \times 0.3}{2 \times 10^5} = 9 \times 10^{-5}$

change in length, $dL = \epsilon_L L = 9 \times 10^{-5} \times 1500 = 0.135\ mm$

Volumetric strain $\epsilon_v = 2\epsilon_c + \epsilon_L = 2 \times 3.825 \times 10^{-4} + 9 \times 10^{-5}$

or $\epsilon_v = 8.55 \times 10^{-4}$

Change in volume, $dV = \epsilon_v \times V = 8.55 \times 10^{-4} \times \frac{\pi}{4} \times 1000^2 \times 1500 = 100.73 \times 10^4\ mm^3$

Pb5) A 0.9m long thin cylinderical shell has 450mm inner diameter and 12 mm thickness. The cylinders is Initially filled with water at atmospheric pressure. Determine the pressure at which an additional water of $187 \times 10^3\ mm^3$ may be pumped into the cylinder, i) ignoring the compressibility of water and ii) Considering compressibility of water.

Take E = 200 GPa and $\gamma = 0.3$ and Bulk modulus $K = 2.1 \times 10^3$ m..

source: diginotes.in

Given

diameter of the cylinder $D = 450\,mm$

Length " " " $L = 0.9\,m = 900\,mm$

thickness " " " $t = 12\,mm$.

Change in volume of the cylinder (= additional water pumped) $dV = 187 \times 10^3\,mm^3$

Bulk modulus of the cylinder $K = 2.1 \times 10^3\,mm^3$

Young's modulus " " " $E = 200\,GPa = 2 \times 10^5\,N/mm^2$

Poisson ratio " " " $\nu = 0.3$.

Circumferential strain

$$\epsilon_c = \frac{\sigma_c}{E} - \frac{\sigma_L}{mE} = \frac{PD}{2tE} - \frac{PD}{4tE} \times \frac{1}{m}$$

$$= \frac{PD}{2tE}\left[1 - \frac{1}{2} \times \frac{1}{m}\right] = \frac{P \times 450}{2 \times 12 \times 2 \times 10^5}\left[1 - \frac{0.3}{2}\right]$$

$$\therefore \epsilon_c = 7.97 \times 10^{-5}\,P$$

Longitudinal strain

$$\epsilon_L = \frac{\sigma_L}{E} - \frac{\sigma_c}{mE} = \frac{PD}{4tE} - \frac{PD}{2tE} \times \frac{1}{m}$$

$$= \frac{PD}{4tE}\left[1 - 2 \times \frac{1}{m}\right] = \frac{P \times 450}{4 \times 12 \times 2 \times 10^5}\left[1 - 2 \times 0.3\right]$$

$$\therefore \epsilon_L = 1.875 \times 10^{-5}\,P$$

Volumetric strain

$$\epsilon_v = 2\epsilon_c + \epsilon_L$$

$$= 2 \times 7.97 \times 10^{-5}\,P + 1.875 \times 10^{-5}\,P$$

$$= 17.82 \times 10^{-5}\,P$$

i) Neglecting compressibility of water:-

The cylinder is initially filled with water at atmospheric pressure. Now an additional water of volume $187 \times 10^3\,mm^3$ is to be pumped into the cylinder at a pressure P.

source: diginotes.in

Therefore the cylinder undergoes an increase in its volume which is equal to its additional water

$$\frac{dV}{V} = \epsilon_v$$

or $dV = V\epsilon_v$

$\therefore \; 187 \times 10^3 = 17.82 \times 10^{-5} P \times \frac{\pi}{4} \times 450^2 \times 900$

or $P = 7.3 \; N/mm^2$

ii) <u>Considering the compressibility of water</u> :-

Additional volume of the water to be pumped into the cylinder is equal to the sum of increase in the volume of cylinder and reduction in the volume of water in the cylinder.

Bulk modulus $K = \frac{P}{\frac{dV}{V}}$

Reduction in volume of water:

$$dV_w = \frac{P}{K} V$$

$$= \frac{P}{2.1 \times 10^3} \times \frac{\pi}{4} \times 450^2 \times 900$$

$\therefore \; dV_w = 68.1 \times 10^3 P \; mm^3$

Additional volume of water $= dV_{cylinder} + dV_{water}$

$187 \times 10^3 = 17.82 \times 10^{-5} \times P \times \frac{\pi}{4} \times 450^2 \times 900 + 68.1 \times 10^3 P$

or $P = 2 \; N/mm^2$

source: diginotes.in

Thick Pressure Vessels :-

* Thick walled pressure vessels are charaterized by considerably lower values of inner radius to wall thickness.

i.e, in thin pressure vessels,

$$\frac{R}{t} \nless 10$$

* They are widely used in the fields of
  - chemical plants,
  - piping,
  - deep submersibles
  - shrunk fit cylinderical components etc.

* As in case of thin cylinders, the thick cylinders are subjected to circumferential stresses and longitudinal stress. Thick spherical vessels are subjected to circumferential stress in their mutually perpendicular diametrical planes, similar to thin spherical vessels.

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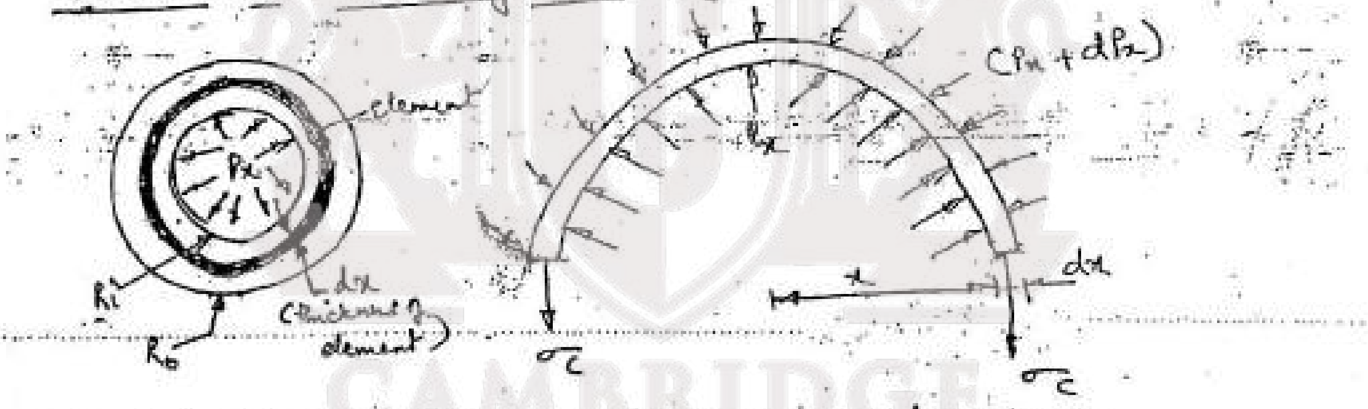
* The thin walled pressure vessels are subjected to negligibly small magnitudes of radial stresses (along the thickness of wall) as their thickness is very small. However the thick pressure vessels are subjected to considerable magnitude of radial stresses, as they have moderately thicker walls.
* Another important difference of thick walled pressure vessels is that the circumferential stress and radial stress vary across the thickness of the wall. But longitudinal stress in case of thick cylinder remains constant.

Stresses in Thick cylinder (Lame's equation) :-



(a) Thick cylinder under internal pressure (b) Element under stress.

* A thick cylinder of length L, inner and outer radii Ri and Ro respectively, subjected to internal pressure P is as shown. Consider a semicircular element of inner radius x and radial thickness dx as shown in figure (b):-
* Let the element be subjected to an internal pressure P_x, external pressure $P_x + dP_x$.
σ_c be the circumferential stress developed in the wall of the section due to applied pressure P.

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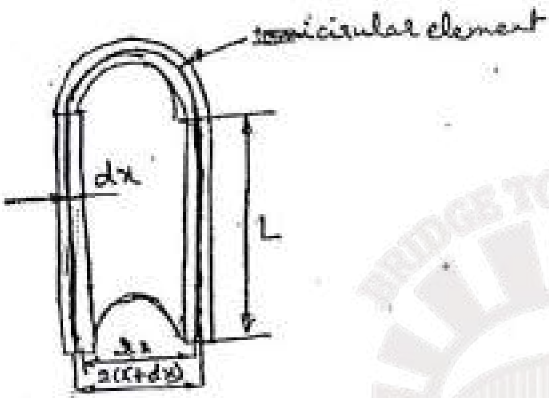
* The element is subjected to the bursting force F_b due to radial pressures P_x and $(P_x + dP_x)$ acting on the projected areas $2xL$ and $2(x+dx)L$ (rectangular areas) respectively.



[Hint
Bursting force in this cylinder $= PLD$

Resisting force $= \sigma_c \, 2tL$]

Now, Bursting force $F_b = P_x(2xL) - (P_x + dP_x)[2(x+dx)L]$

Resisting force, $F_R = \sigma_c \times 2\,dx\,L$

For equilibrium of the element, $F_b = F_R$.

$P_x(2xL) - (P_x + dP_x)[2L(x+dx)] = \sigma_c \, 2dxL$

$P_x 2x - P_x 2x - P_x dx - dP_x 2x - dP_x 2dx = \sigma_c dx$

$xP_x - xP_x - P_x dx - x\,dP_x - dP_x dx = \sigma_c dx$.

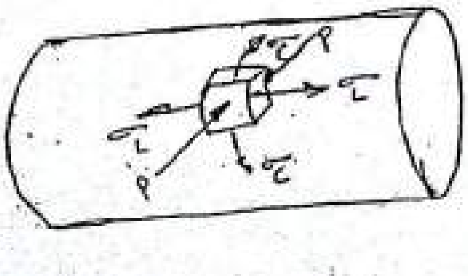
Neglecting higher order term $dP_x dx$ we get

$$\sigma_c dx = -x\,dP_x - P_x dx$$

or $\sigma_c + x\dfrac{dP_x}{dx} + P_x\dfrac{dx}{dx} = 0$

$\therefore \sigma_c + x\dfrac{dP_x}{dx} + P_x = 0$ ——— (1)

* An element in the wall of the cylinder is subjected to
i) Radial pressure P_x
ii) Circumferential stress σ_c and
iii) Longitudinal stress σ_L



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Thick and Thin cylinders

* Cylindrical shells are used to store or transport oil, petroleum products, gas, water etc.
* For example, boilers, water tanks etc.
* When the shells are filled with a fluid, their walls are subjected to stresses.

Cylinders are considered to be thin when their wall thickness is less than $\frac{1}{20}^{th}$ of their diameter.

Cylinders are considered to be thick when their wall thickness is more than $\frac{1}{20}^{th}$ of their diameter.

Thin cylinders

Thin cylinders are subjected to two types of stresses at the surface.

(i) Circumferential / hoop stress which acts along the circumference of the cylinder.

(ii) Longitudinal stress (σ_L) which act along the length of the cylinder.

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5. A solid round bar 4m long and 50mm in diameter was found to extend by 4.6 mm long under a tensile load of 50kN. This bar is used a strut with both ends hinged(pinned) Determine Euler's crippling load for the bar and also safe load taking factor of safety as 4. (Dec 2014)

6. Find the Euler's critical load for a hollow cylindrical cast iron column 150 mm external diameter, 20 mm wall thickness if it is 6 m long with hinges at both ends. Assume Young's modulus of cast iron as 80 kN/mm^2. Compare this load with given by Rankine's formula using Rankine's constant $a = 1/1,600$ and $f_c = 567$ N/mm^2. (June 2015)

Internals Pattern

Max marks = 40
5 - Parts
each part has 2 Questions, can answer any one
Total given questions = 10
Questions to be answered = 5
each Question carries 8 marks
⇒ 5 × 8 = 40 marks
Q M

Final IA marks (20) marks = avg of best 2 IA = avg / 2

External Max marks = 80
5 Parts
each part - 16 marks.

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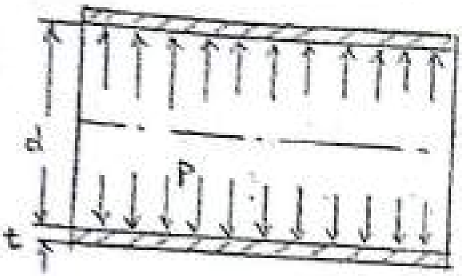
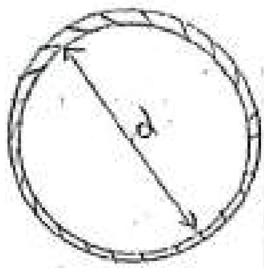
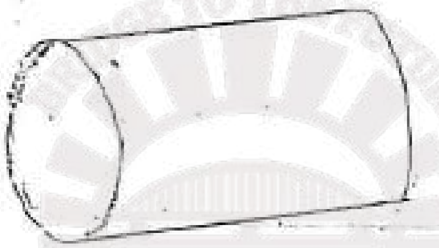
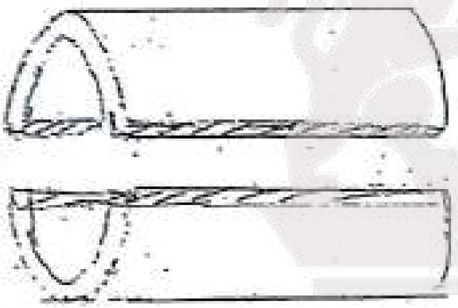
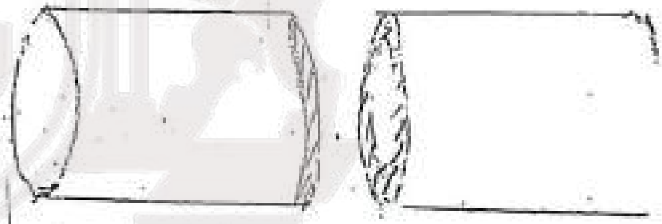








Circumferential failure



Longitudinal failure

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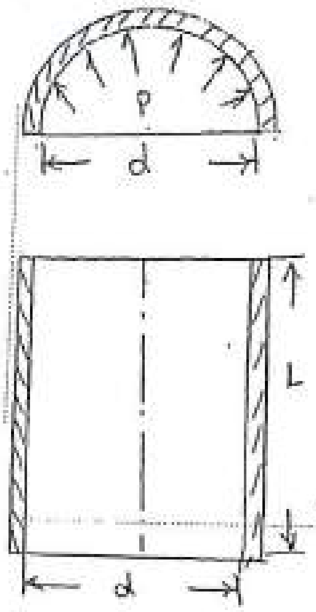
1. Circumferential Stress



Load = Bursting force

$= p \times d \times L$

Intensity of pressure × projected area on which p is acting

$\sigma_c \rightarrow$ Circumferential Stress

$\sigma_h \rightarrow$ hoop stress

$\therefore \sigma_c \times$ Resisting area

$\sigma_c \times [tL + tL] = \sigma_c(2tL)$

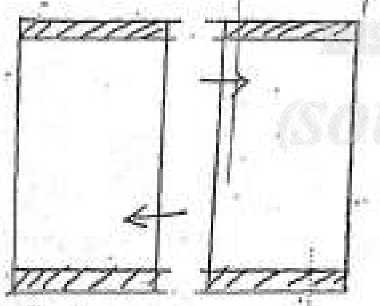
$p \times d \times L = \sigma_c (2 \times t \times L)$

$\sigma_c = \dfrac{p \times d \times L}{2 \times t \times L}$

$$\boxed{\sigma_c = \frac{pd}{2t}}$$

$\therefore \sigma_c = \sigma_h$

2. Longitudinal Stress



d = mean dia

Bursting force $= p \times \dfrac{\pi d^2}{4}$ (c/s area)

Resisting force $= \sigma_L \times \pi \times d \times t$

$\therefore p \times \dfrac{\pi d^2}{4} = \sigma_L \, \pi d t$

$$\boxed{\sigma_L = \frac{pd}{4t}}$$

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source: diginotes.in

Change in diameter

The stress along circumferential dir^n is $\perp^{ar}$ to the stress in longitudinal direction.

Hence, the circumferential strain ε_h is given by

$$\varepsilon_h = \frac{\delta d}{d} = \frac{\sigma_h}{E} - \frac{\nu \sigma_L}{E}$$

$$= \frac{Pd}{2tE} - \frac{\nu Pd}{4tE} = \frac{Pd}{2tE}\left(1 - \frac{\nu}{2}\right) = \frac{Pd}{2tE}\left(\frac{2-\nu}{2}\right) = \frac{Pd}{4tE}(2-\nu)$$

$$\varepsilon_h = \frac{\delta d}{d} = \frac{Pd}{4tE}(2-\nu) \quad\quad \boxed{\delta d = \frac{Pd^2}{4tE}(2-\nu)}$$

Change in length

$$\varepsilon_L = \frac{\delta L}{L} = \frac{\sigma_L}{E} - \nu\frac{\sigma_h}{E}$$

$$\frac{\delta L}{L} = \frac{Pd}{4tE} - \frac{\nu Pd}{2tE} = \frac{Pd}{2tE}\left(\frac{1}{2} - \nu\right) = \frac{Pd}{2tE}\left(\frac{1-2\nu}{2}\right)$$

$$\varepsilon_L = \frac{\delta L}{L} = \frac{Pd}{4tE}(1-2\nu) \quad\quad \boxed{\delta L = \frac{PdL}{4tE}(1-2\nu)}$$

Change in volume

$$V = \frac{\pi}{4} d^2 L$$

$$\delta V = \frac{\pi}{4}\left(2d\,\delta d\,L + d^2 \delta L\right)$$

$$\therefore \frac{\delta V}{V} = \frac{\frac{\pi}{4}\left(2d\,\delta d\,L + d^2 \delta L\right)}{\frac{\pi}{4} d^2 L}$$

$$\therefore \frac{\delta V}{V} = \frac{2\,\delta d}{d} + \frac{\delta L}{L}$$

$$\varepsilon_V = 2\varepsilon_h + \varepsilon_L$$

$$\varepsilon_V = 2\left(\frac{Pd}{4tE}\right)(2-\nu) + \frac{Pd}{4tE}(1-2\nu)$$

$$\therefore \boxed{\delta V = \frac{Pd}{4tE}(5-4\nu)V}$$

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A thin cylindrical shell 1.2m in diameter and 3m long has a metal wall thickness of 12mm. It is subjected to an internal fluid pressure of 3.2 MPa. Find the circumferential and longitudinal stress in the wall. Determine change in length, diameter and volume of the cylinder. Assume E = 210 GPa and $\nu = 0.3$

→ $d = 1.2m$

$l = 3m$

$t = 12mm = 12 \times 10^{-3} m$

$P = 3.2 \times 10^{6} N/m^2$

$E = 210 GPa$

$\nu = 0.3$

(i) $\frac{\delta L}{L} = \frac{Pd}{4tE}(1-2\nu)$

$\delta L = \frac{Pd}{4tE}(1-2\nu)L$

$= \frac{3.2 \times 10^{6} \times 1.2 \times (1-2(0.3)) \times 3}{4 \times 12 \times 10^{-3} \times 210 \times 10^{9}}$

$= 4.571 \times 10^{-4} m$

(ii) $\frac{\delta d}{d} = \frac{Pd}{4tE}(2-\nu)$

$\delta d = \frac{Pd^2}{4tE}(2-\nu)$

$= \frac{3.2 \times 10^{6} \times (1.2)^2 (2-0.3)}{4 \times 12 \times 10^{-3} \times 210 \times 10^{9}}$

$= 7.771 \times 10^{-4} m$

(iii) $\frac{\delta V}{V} = \frac{Pd}{4tE}(5-4\nu)$

$V = \frac{\pi d^2}{4} L$

$\delta V = \frac{PdV}{4tE}(5-4\nu)$

$= \frac{3.2 \times 10^{6} \times 1.2 \times (3.39)(5-4(0.3))}{4 \times 12 \times 10^{3} \times 210 \times 10^{9}}$

$= 4.90 \times 10^{-7} m^3$

$\sigma_c = \frac{Pd}{2t} = \frac{3.2 \times 10^{6} \times 1.2}{2 \times 12 \times 10^{-3}} = 160 MPa$

$\sigma_L = \frac{Pd}{4t} = 80 MPa$

source: diginotes.in

2. A thin cylinder, 2m long and 200 mm in diameter with 10mm thickness is filled completely with a fluid, at the atmospheric pressure. If an additional 25000 mm^3 fluid is pumped in, find the longitudinal and hoop stress developed. Also determine the changes in diameter and length if $E = 2\times10^5\ N/mm^2$ and poissons ratio = 0.3

→ $L = 2m$

$d = 200\times10^{-3}\ m$

$t = 10\times10^{-3}\ m$

$E = 2\times10^5\ N/mm^2 = 2\times10^5\times10^6\ N/m^2$

$\nu = 0.3$

$\delta V = 25000\ mm^3$

$= 25000\times10^{-9}\ m^3$

$V = \frac{\pi d^2}{4}L = \frac{\pi(200\times10^{-3})^2\times 2}{4}$

$= 0.062\ m^3$

$\therefore \delta V = \frac{Pd}{4tE}(5-4\nu)V$

$25000\times10^{-9} = \frac{P\times200\times10^{-3}}{4\times10\times10^{-3}\times2\times10^{11}}(5-4(0.3))(0.06$

$P = 4.2\ MPa$

$\therefore \sigma_h = \frac{Pd}{2t} = \frac{4.2\times10^6\times200\times10^{-3}}{2\times10\times10^{-3}}$

$= 42\ MPa$

$\sigma_L = \frac{Pd}{4t} = 21\ MPa$

$\delta d = \frac{Pd^2}{4tE}(2-\nu) = \frac{4.2\times10^6\times(200\times10^{-3})^2(2-0.3)}{4\times10\times10^{-3}\times2\times10^{11}} = 3.57\times10^{-5}\ m$

$\delta L = \frac{PdL}{4tE}(1-2\nu) = \frac{4.2\times10^6\times(200\times10^{-3})2}{4\times10\times10^{-3}\times2\times10^{11}}(1-2(0.3)) = 8.4\times10^{-5}\ m$

source: diginotes.in

3. The diameter of a city water supply pipe is 750mm. It has to withstand a water head of 50m. Find the thickness of the pipe, if the permissible stress is 20 N/mm². Take unit weight of water as 9810 N/m³

→ pressure of water $= wh = 9810\ N/m^3 \times 50m$

$= 490500\ N/m^2$

$\therefore \sigma = \frac{Pd}{2t}$

$t = \frac{Pd}{2\sigma} = \frac{490500 \times 750 \times 10^{-3}}{2 \times 20 \times 10^6} = 0.00919m = 9.19mm$

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Thick Cylinders

In thick cylindrical shells, the circumferential stress varies from maximum value at the inner surface to a non-zero minimum value at the outer surface.

Radial stress induced in the thin walled pressure vessels is negligibly small as the wall thickness is very small, when compared to its radius.

The radial pressure (stress) varies from a maximum value at the inner surface to a minimum value at the outer surface.

The variation of circumferential stress and radial pressure along the thickness of the wall is obtained using Lame's theory.

Lame's Theory

is based on the following assumptions.

1) The shell is made up of homogeneous and isotropic material.
2) The longitudinal strain is constant. It is independent of radius of the shell.

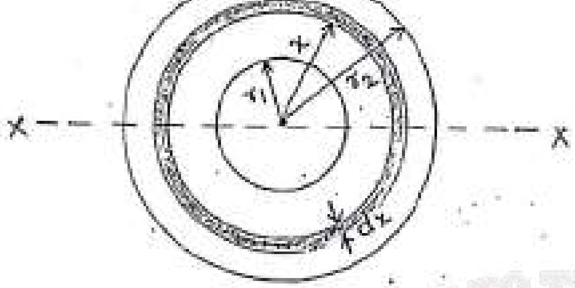
Consider a thick cylindrical shell of length L, internal radius r_1, and external radius r_2.

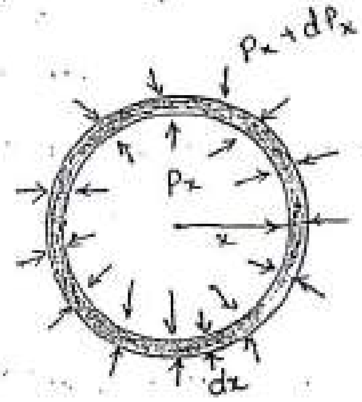
Consider an elementary ring of radius x and thickness dx.

Let P_x = Radial pressure on inner surface of ring and $P_x + dP_x$ = Radial pressure on outer surface

source: diginotes.in

of ring.





For section x-x;

Bursting force $= P_x(2x)L - (P_x + dP_x)2(x+dx)L$

$= 2xLP_x - 2L[(P_x + dP_x)(x+dx)]$

$= 2xLP_x - 2L[xP_x + P_x dx + x dP_x + dP_x dx]$

$= 2xLP_x - 2LxP_x - 2LP_x dx - 2Lx dP_x - 2L dP_x dx$

$= -2LP_x dx - 2Lx dP_x - 2L dP_x dx$

$= -2LP_x dx - 2Lx dP_x$

Resisting force $= \sigma_x (2dx)L$

$\therefore$ For equilibrium, resisting force = bursting force

$$\sigma_x (2dx)L = -2LP_x dx - 2Lx\, dP_x$$

$$\sigma_x = -\frac{2LP_x dx}{2dxL} - \frac{2Lx\, dP_x}{2dxL}$$

$$\sigma_x = -P_x - x\frac{dP}{dx} \quad \text{—(1)}$$

source: diginotes.in

WKT, There are three stresses acting in three mutually perpendicular directions:

i) Radial pressure, P_x which is compressive

ii) Circumferential stress σ_x which is tensile

iii) Longitudinal stress σ_L which is tensile

From Generalised Hooke's Law

$$\varepsilon_L = \frac{\sigma_L}{E} - \frac{\nu \sigma_x}{E} - \left(\frac{-\nu P_x}{E}\right)$$

As longitudinal strain = constant

As ε_L is constant, σ_L is also constant

$$\therefore \frac{-\nu \sigma_x}{E} + \frac{\nu P_x}{E} = \text{constant}$$

$$-\sigma_x + P_x = \text{constant}$$

$$\therefore \sigma_x - P_x = \text{constant}$$

$$\sigma_x - P_x = 2A$$

$$\sigma_x = P_x + 2A \quad \text{---} (2)$$

From eqn (1) & (2)

$$P_x + 2A = -P_x - x\frac{dP}{dx}$$

$$P_x + P_x + 2A = -x\frac{dP}{dx}$$

$$2(P_x + A) = -x\frac{dP}{dx}$$

$$-2\frac{dx}{x} = \frac{dP}{P_x + A}$$

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Integrating,

$$\log_e(P_x + A) = -2\log_e x + \log_e B$$

$$\log_e(P_x + A) = \log_e\left[\frac{B}{x^2}\right]$$

$$P_x + A = \frac{B}{x^2}$$

$$\boxed{P_x = \frac{B}{x^2} - A} \quad \text{--- (3)}$$

Substituting P_x in eqn (2)

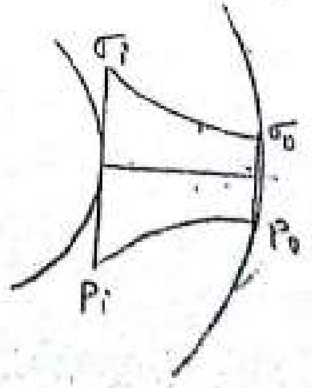
$$\sigma_x = P_x + 2A$$

$$\sigma_x = \frac{B}{x^2} - A + 2A$$

$$\boxed{\sigma_x = \frac{B}{x^2} + A} \quad \text{--- (4)}$$

eqn (3) & (4) are known as Lame's equation

The variation of P_x and σ_x with radial distance x



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1. A thick cylinder with internal diameter 80 mm and external diameter 120mm is subjected to an external pressure of 40 kN/m² when the internal pressure is 120 kN/m². Calculate the circumferential stress at external and internal surfaces of the cylinder. Plot the variation of circumferential stress and radial pressure on the thickness of the cylinder.

→ From Lame's equations,

$$P_x = \frac{B}{x^2} - A \quad \text{—(1)}$$

$$\sigma_x = \frac{B}{x^2} + A \quad \text{—(2)}$$

at $(d_0 = 120\text{ mm})$ or $(x_0 = 60\text{ mm})$ $\quad P_0 = 40\times10^3\ N/m^2$

at $d_i = 80\text{ mm}$ or $(x_i = 40\text{ mm})$ $\quad P_i = 120\times10^3\ N/m^2$

$$\therefore P_0 = \frac{B}{x_0^2} - A, \quad 40\times10^3 = \frac{B}{(60\times10^{-3})^2} - A \quad \text{—(3)}$$

$$P_i = \frac{B}{x_i^2} - A; \quad 120\times10^3 = \frac{B}{(40\times10^{-3})^2} - A \quad \text{—(4)}$$

Solving (3) & (4)

$B = 230.4$

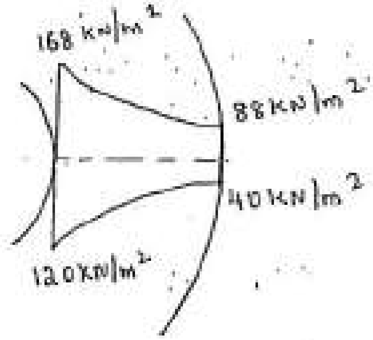
$A = 24000$

$$\therefore \sigma_{x_0} = \frac{B}{x_0^2} + A, \quad \sigma_{x_0} = \frac{230.4}{(60\times10^{-3})^2} + 24000 = 88000 = 88\ kN/m^2$$

$$\sigma_{x_i} = \frac{B}{x_i^2} + A, \quad \sigma_{x_i} = \frac{230.4}{(40\times10^{-3})^2} + 24000 = 168000 = 168\ kN/m^2$$

source: diginotes.in

Variation of Circumferential stress σ_x and radial pressure p_x.



2. A pipe of 400 mm internal diameter and 100 mm thickness contains a fluid at a pressure of 80 N/mm². Find the maximum and minimum hoop stresses across the section. Also sketch radial and hoop stresses distribution across the section.

→ The radial pressure is given by

$$p_x = \frac{B}{x^2} - A \quad —①$$

Given: $x_i = 200$ mm t = 100 mm $x_o = x_i + t = 200 + 100$

$x_o = 300$ mm

$p_i = 80$ N/mm² $p_o = 0$

$$p_o = \frac{B}{x_o^2} - A \;, \quad 0 = \frac{B}{(300\times10^{-3})^2} - A \quad —②$$

$$p_i = \frac{B}{x_i^2} - A \;, \quad 80 = \frac{B}{(200\times10^{-3})^2} - A \quad —③$$

Solving ① & ②

B = 5.76

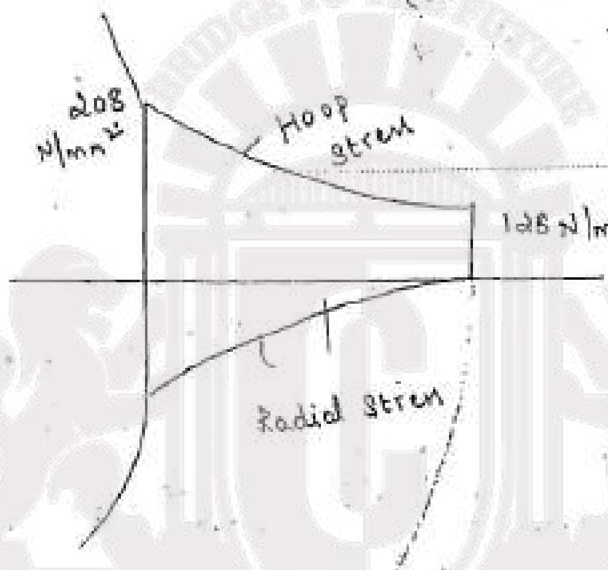
A = 64

source: diginotes.in

$\therefore \quad \sigma_x = \frac{B}{x^2} + A$

$\sigma_{x_0} = \frac{B}{x_0^2} + A$, $\quad \sigma_{x_0} = \frac{5.76}{(300 \times 10^{-3})^2} + 64 = 128 \ N/mm^2$

$\sigma_{x_i} = \frac{B}{x_i^2} + A$. $\quad \sigma_{xi} = \frac{5.76}{(200 \times 10^{-3})^2} + 64 = 208 \ N/mm^2$



3. A thick cylinder of internal diameter 160 mm is subjected to an internal pressure of 40 N/mm². If the allowable stress in the material is 120 N/mm², find the required wall thickness of the cylinder.

→ Given: $x_i = 80$ mm $\qquad P_i = 40 \ N/mm^2$

$t = ?$ $\qquad \sigma_{x_i} = 120 \ N/mm^2$

source: diginotes.in

$$P_{x_i} = \frac{B}{x_i^2} - A \qquad \sigma_{x_i} = \frac{B}{x_i^2} + A$$

$$40 = \frac{B}{(80)^2} - A \quad -(1) \qquad 120 = \frac{B}{80^2} + A \quad -(2)$$

Solving (1) & (2)

$B = 512000$

$A = 40$

At the outer surface of the shell, $P_x = 0$ ie P_{x_o}

$$P_{x_o} = \frac{B}{x_o^2} - A$$

$$0 = \frac{B}{x_o^2} - A \quad , \quad 0 = \frac{512000}{x_o^2} - 40$$

$$40 = \frac{512000}{x_o^2}$$

$$x_o^2 = \frac{512000}{40}$$

$$x_o = 113.14 \text{ mm}$$

$\therefore x_o = x_i + t$

$t = x_o - x_i = 113.14 - 80$

$\therefore t = 33.14 \text{ mm}$

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source: diginotes.in

4. A thick cylindrical shell of 200 mm internal diameter is subjected to an internal fluid pressure of 7 N/mm². If the permissible tensile stress in the shell material is 8 N/mm², find the thickness of the shell.

→ Given; $x_i = 100$ mm, $P_{x_i} = 7$ N/mm²

$t = ?$ if $\sigma_{x_i} = 8$ N/mm²

$$P_{x_i} = \frac{B}{x_i^2} - A \quad , \quad 7 = \frac{B}{(100)^2} - A \quad -(1)$$

$$\sigma_{x_i} = \frac{B}{x_i^2} + A \quad , \quad 8 = \frac{B}{(100)^2} + A \quad -(2)$$

∴ Solving (1) & (2)

$B = 75000$

$A = 0.5$

at $P_{x_0} = 0$

$$P_{x_0} = \frac{B}{x_0^2} - A$$

$$0 = \frac{75000}{x_0^2} - 0.5$$

$$x_0^2 = \frac{75000}{0.5}$$

$x_0 = 387.3$ mm

$t = x_0 - x_i = 387.3 - 100$

∴ $t = 287.3$ mm

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Determine i) circumferential strains at the inner and outer surfaces and ii) Longitudinal strains at the inner and outer surfaces and prove that the longitudinal strain is constant through the cylinder. Take E = 200Gpa and $\nu = 0.3$. [$r_i = 150mm$, $r_o = 185$ mm, $P_i = 10MPa$, $P_o = 0$

$\sigma_{x_i} = 48.4$ MPa (tension) $\sigma_{x_o} = 38.4$ MPa (tension).

Equilibrium of cylinder in transverse plane

$$\pi d_i^2 P = \pi (R_o^2 - R_i^2) \sigma_L$$

$$\frac{\pi}{4} \times 150^2 \times 10 = \pi (185^2 - 150^2) \sigma_L$$

$$\sigma_L = 19.2 MPa$$

inner surface

$$\varepsilon_{ci} = \frac{\sigma_{ci}}{E} - \frac{\mu \sigma_L}{E} + \frac{\mu P_i}{E} = \frac{48.4}{2\times10^5} + \frac{10\times0.3}{2\times10^5} - \frac{19.2\times0.3}{2\times10^5}$$

$$= 22.82 \times 10^{-5}$$

$$\therefore \varepsilon_{Li} = \frac{\sigma_L}{E} + \frac{\mu P_i}{E} - \frac{\sigma_{ci}\mu}{E} = \frac{19.2}{2\times10^5} + \frac{10\times0.3}{2\times10^5} - \frac{48.4\times0.3}{2\times10^5} = 3.84\times10^{-5}$$

Outer surface

$$\varepsilon_{co} = \frac{\sigma_{co}}{E} - \frac{\sigma_L \mu}{E} = \frac{38.4}{2\times10^5} - \frac{19.2\times0.3}{2\times10^5} = 16.32\times10^{-5}$$

$$\varepsilon_{Lo} = \frac{\sigma_L}{E} - \frac{\sigma_{co}\mu}{E} = \frac{19.2}{2\times10^5} - \frac{38.4\times0.3}{2\times10^5} = 3.84\times10^{-5}$$

Longitudinal strain is constant through the cylinder,

Since $\varepsilon_{Li} = \varepsilon_{Lo}$

source: diginotes.in

percentage error involved when the thickness is calculated based on thin vessel theory.

ie 't_L' calculated from Lame's theory

and 't' from $\sigma_c = \frac{Pd}{2t}$

$$\%\ error = \frac{t_L - t}{t} \times 100 =$$

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The moment causing a portion of the beam to be bent into concave shape on its top surface is known as sagging moment or positive moment, whereas the moment tending to bend a beam with convex shape on its top is known as Hogging moment or negative moment.

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Shear Force & Bending Moment in Beams.

Beam: A beam is a long structural member with relatively small - cross - sectional dimensions and subjected to vertical loads or bending due to normal or transverse forces acting on it.

Beam with, length (L), depth (D) & width (W).

Note: L >>> W & D

✓ Beams are normally horizontal & subjected to vertical loads

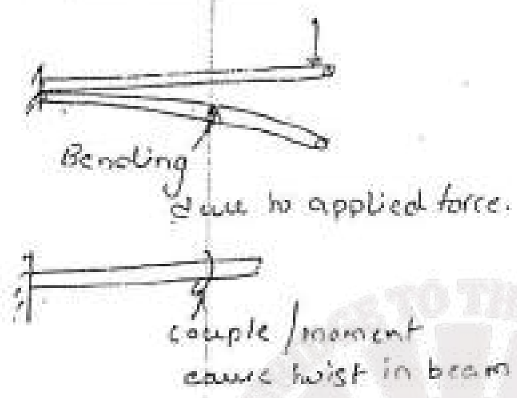
✓ Applications of beam: Used to support floors, cealings of the buildings.
- pipes carrying water
- shafts supported on bearings
- lathe beds
- fans with shaft - fixed to cealing
- Bridges, flyover, [illegible]
- Electric poles
- Crane hooks
- Brake lever
- Screw jack handle
- Building frame
- Arches
- Ladder . etc

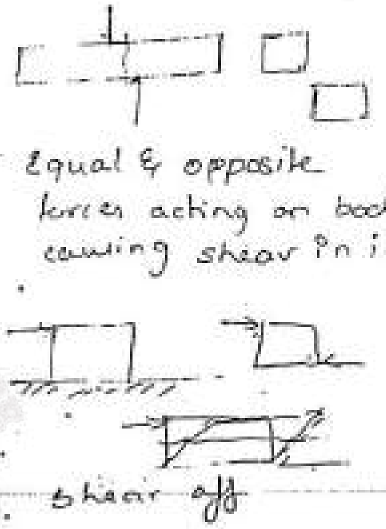
✓ A system of external loads which acts on beams are always "right angles to its axis".

✓ Couples acting in a plane passing through the axis of the beam is known as "Bending".

source: diginotes.in

1. Forces which tries to shear off or tear the object or beam are known as shear forces.





Types of Beam, Loads & Supports.

⇒ Types of BEAMS

1) Straight & Curved beam

- A beam whose axis is along a straight line

Ex: Building frame, Electric pole, Brake lever, screw jack handle etc.

- If the axis of beam is curved then it is called curved beam

Ex: Orches, chain hooks, crane hooks etc

2) Horizontal, Vertical & Inclined beam.

- If the axis of the beam is straight & horizontal, it is called Horizontal beam.

Ex: Building frame, beams of bridges etc

- If the axis of the beam is straight & Vertical, then it is called Vertical beam.

Ex: Electric pole.

- If the axis of the beam is straight & Inclined, then it is called Inclined beam

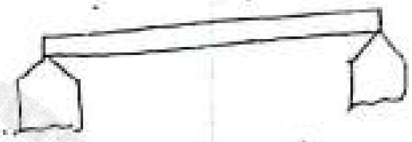
Ex: Ladder.

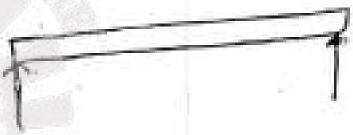
source: diginotes.in

3) Cantilever, simply supported, fixed, overhanging and continuous beam:

Cantilever beam : A beam fixed @ one end & free @ the other end is known as cantilever beam.

Simply supported beam :
In a simply supported beam, the beam rests freely on the supports @ its two ends.





↳ Hinged support.
It is a type of simply support beam, as shown in the figure.
⇒ Allows beam to rotate about its support.

Beam with Hinges.

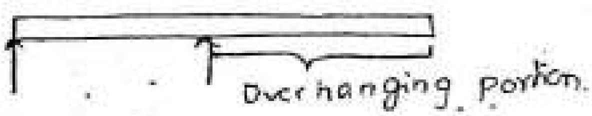
↳ Roller supported beam.
If the beam is supported on rollers it is called Roller Support.
⇒ Free to move along its support
⇒ Can rotate about the support

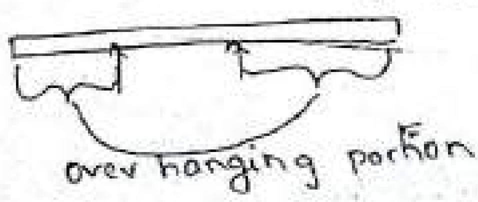
Beam with Rollers

Fixed beam: If the both ends of the beams are fixed or built in walls, it is called a fixed beam.
⇒ Beam end is not free to translate or rotate

Beams is rigidly fixed

Overhanging beam: If the end portion of the beam is extended beyond the support, it is called a "overhanging beam".





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Continuous beam:

A beam supported by more than two supports is called as continuous beam.

Propped beam:

It is a beam with one end fixed & the other end simply supported. It is also called as propped cantilever.

Beam with one end Hinged & the other on Rollers.

If one end of a beam is hinged & the other end is on rollers, the beam can resist load in any direction.

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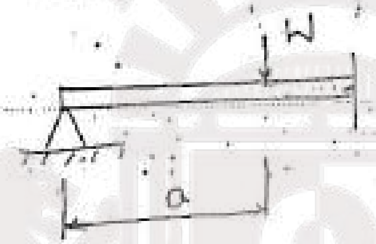
Types of load.

(i) Point or concentrated load.

A load which is assumed to act @ a point is called point or concentrated load. Point load denoted as "W" & expressed as N or KN.

Simply supported beam with point load 'W' @ 'C'

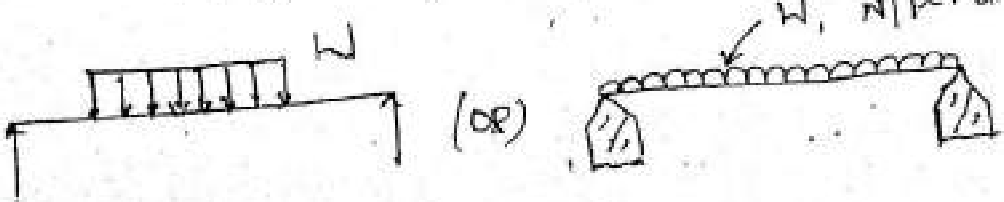


2) Uniformally Distributed Load (UDL)

If the loads acts on the beam spread over some area, it is called distributed load. The rate of loading is not uniform here:

Rate of loading is not uniform ⟺

UDL: A uniformly distributed load is one which is spread over a beam in such a manner that the rate of loading 'W' is uniform along the length



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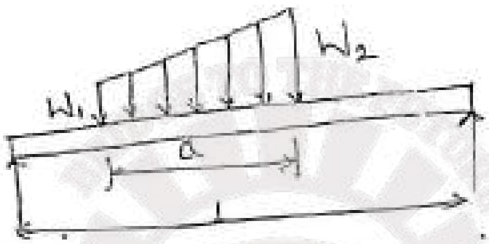
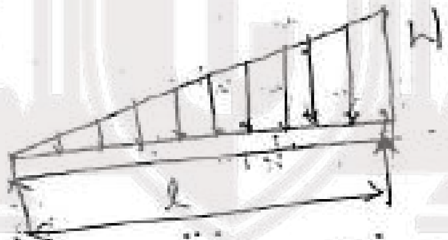
U.V.L - Uniformly varying load

If the load is spread over a beam in such a manner that the rate of loading varies from point to point along the beam, it is called uniformly varying load.



When the load is zero @ one end & increases uniformly to the maximum @ the other end, then the load is known as Triangular load.



Couple : A beam may also be subjected to a couple as shown below.

Note: No load only moment.

source: diginotes.in

Procedure for Drawing SF & BM Diagrams

Steps involved in drawing the shear force & bending moment diagrams for statically determinate beams.

- Determine the reaction forces & reaction moments using the equilibrium equations

$$\Sigma F = 0 \quad , \quad \Sigma M = 0$$

- Determine the shear force @ all the salient points such as supports, points of application of load & other points of intrest.

- Draw the shear force diagram using the relation $\frac{dF}{dx} = -w$ & locate the section @ which the shear force changes its sign, if it does so.

- Find the magnitudes of B.M @ all the salient point & the values of maximum & minium B.M are to be found @ the sections which are subjected to zero shear force

Draw the B.M diagram using the equation

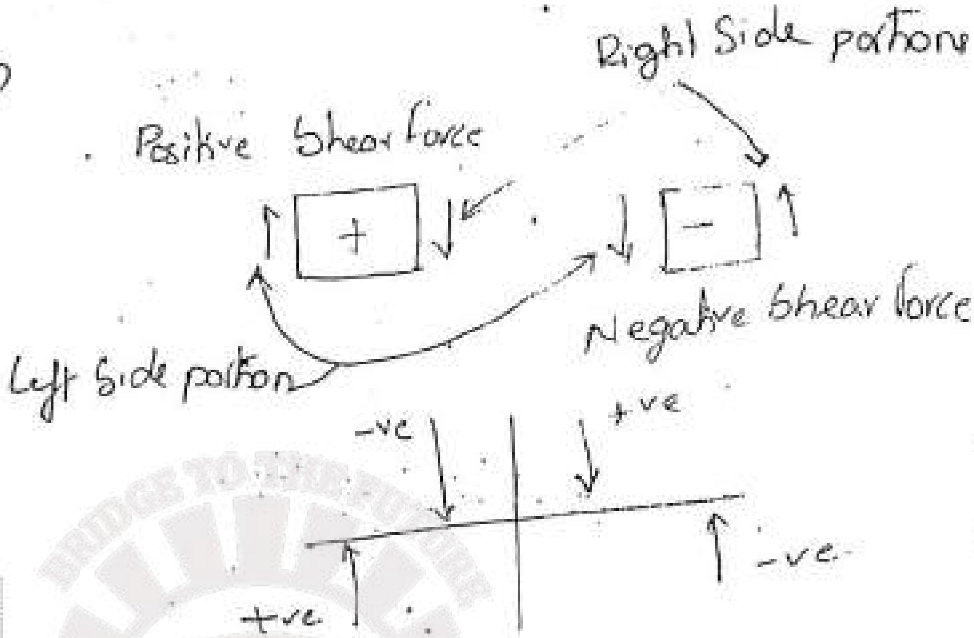
$$F = \frac{dM}{dx}$$

- Locate the point of contraflexure if any. Point of contraflexure is a section @ which change in sign of bending moment occurs.

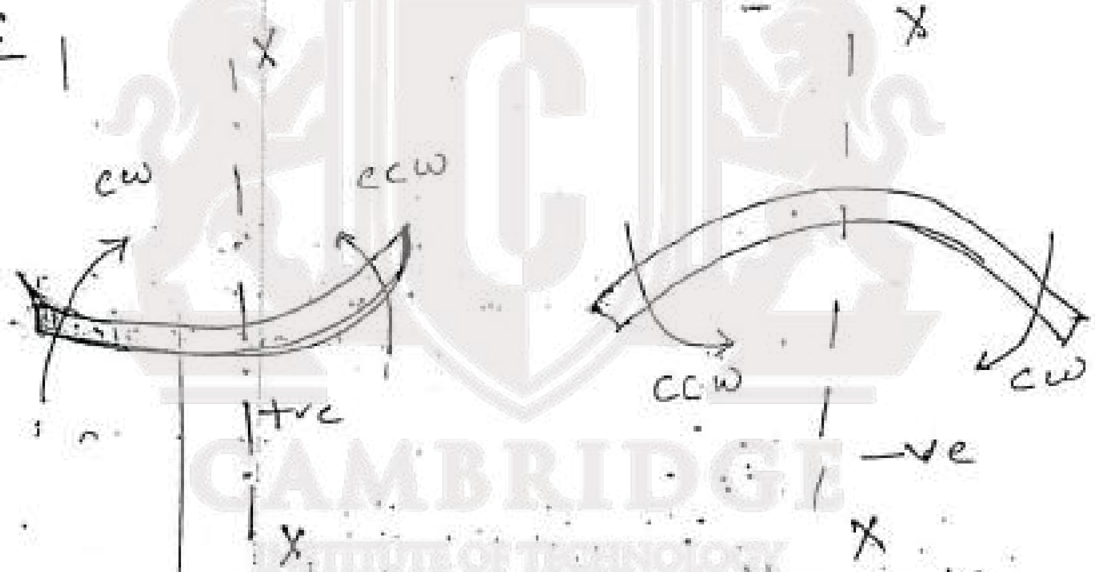
source: diginotes.in

Sign Convention

Shear force:



BM:



Sagging Moment

Hogging Moment

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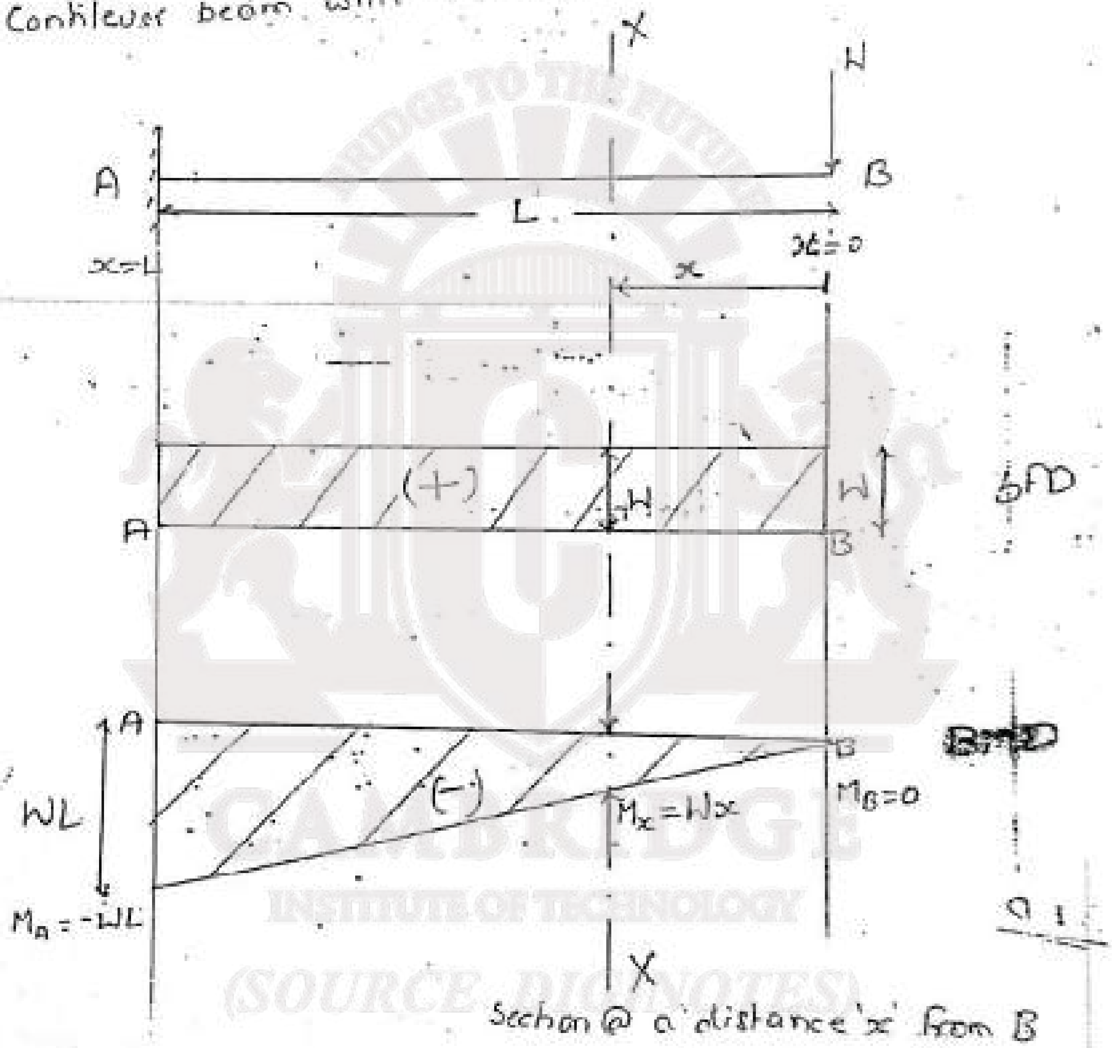
SFD & BMD for different types of beams subjected to different types of loads.

1) CANTILEVER BEAM

a) Cantilever beam with Point load



Section @ a distance 'x' from B

Shear Force Diagram

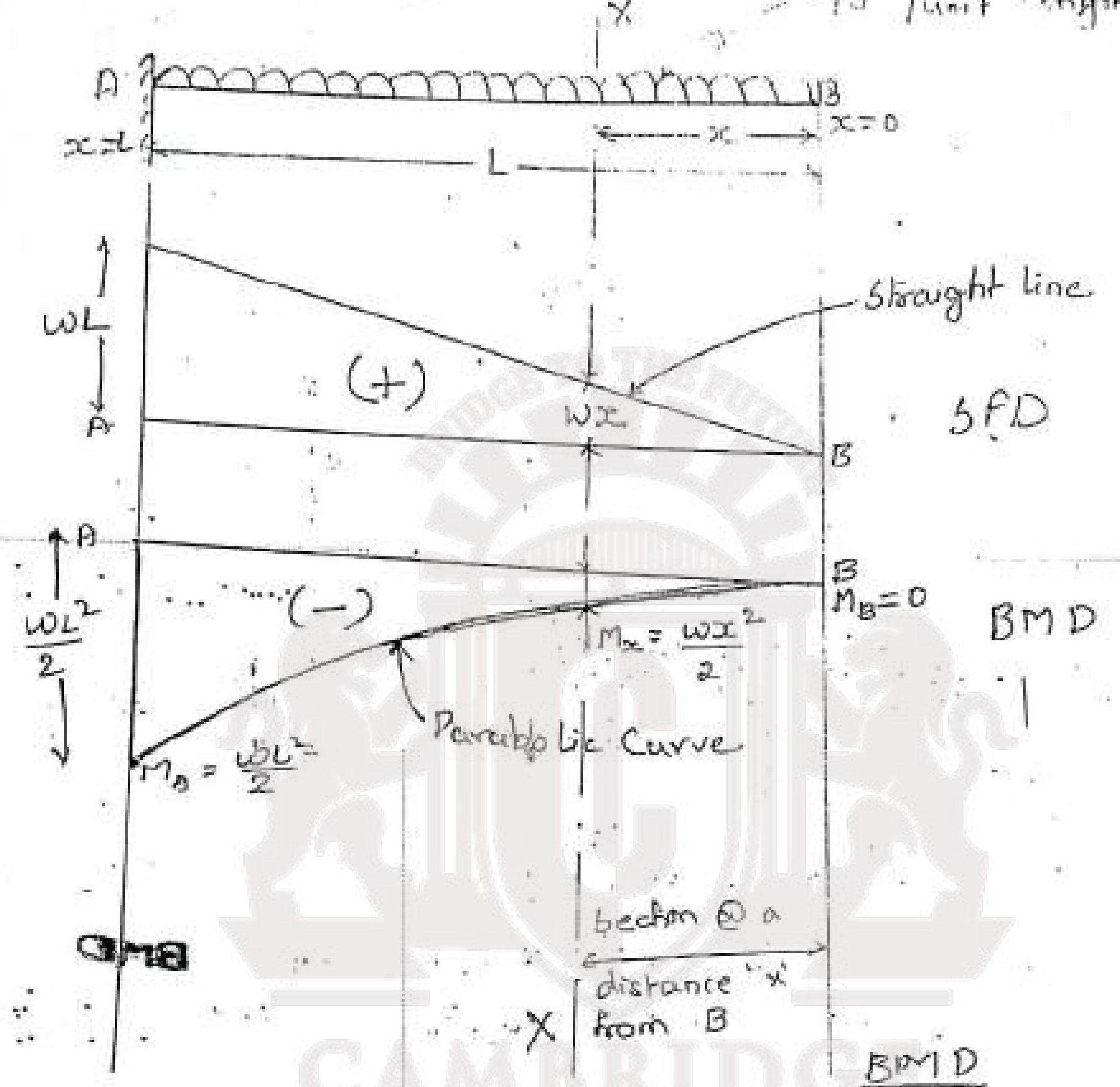
- when $x=0$, SF @ B = +W
- when $x=L$, SF @ A = +W
- SF remains constant @ all sections.
- +ve ⇒ Moves in upward direction

Bending Moment Diagram

- when $x=0$, BM @ B = 0
- when $x=L$, BM @ A = −WL
- −ve because bending is Hogging
- BM form of straight line & varies from point to point

source: diginotes.in

SFD & BMD for a C.B with UDL
w/unit length



SFD

UDL = W per unit length

SF @ X-X = + wx

+ve SF since slides down (or) CW

$F_x = wx$ ∴ equation of a straight line

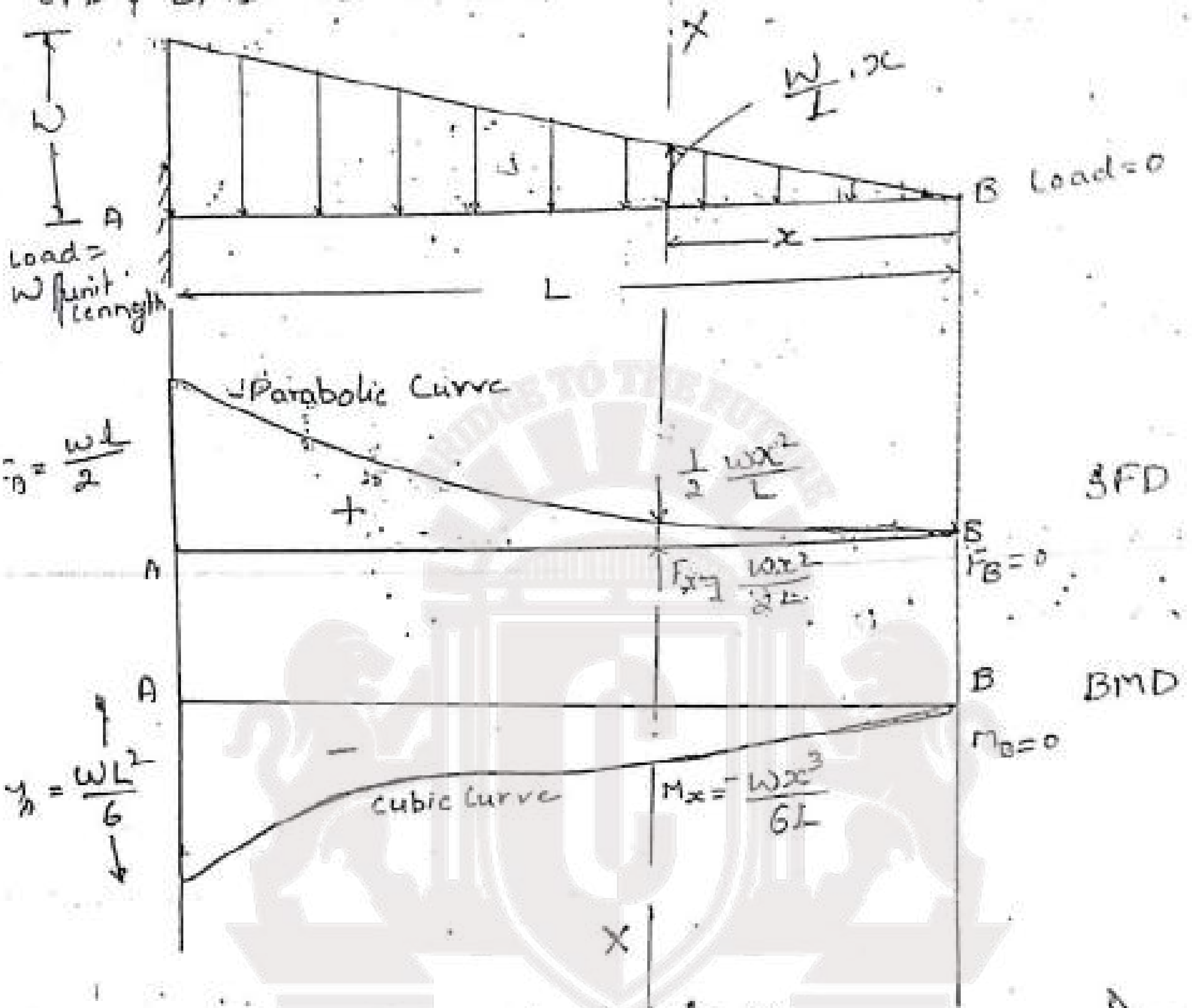
- when x = 0, SF @ B = 0
- when x = L, SF @ A = WL

BMD

- BM @ X-X = − Load x distance
  $M_x = -(wx) \times \frac{x}{2} = -\frac{wx^2}{2}$
- $M_A = -\frac{wL^2}{2}$
- When x = 0, BM @ B = 0
- when x = x, BM @ X = $-\frac{wx^2}{2}$
- when x = L, BM @ A = $-\frac{wL^2}{2}$
- −ve represents Hogging
- Equation represents "Parabola"

source: diginotes.in

SFD & BMD for a cantilever beam with a UVL



. Rate of loading for the length 'L' = W

∴ Rate of loading for the length 'x' = ? ⇒ $\frac{Wx}{L}$

. SF @ B = 0

SF @ XX = Area of loading diagram between "X-X & B"

$= +\frac{1}{2}(x)\left(\frac{Wx}{L}\right) = \frac{Wx^2}{2L}$

SF @ A = $\frac{1}{2} \times L \times \left(\frac{WL}{L}\right) = \frac{WL}{2}$

∴ when $x = 0$, SF @ B = 0

when $x = L$, SF @ A = $\frac{WL^2}{2L} = \frac{WL}{2}$

BM @ B, when $x=0 \Rightarrow M_B = 0$

BM @ X-X = load × distance

$= -\left(\frac{Wx^2}{2L}\right) \times \frac{x}{3} = \frac{-Wx^3}{6L}$

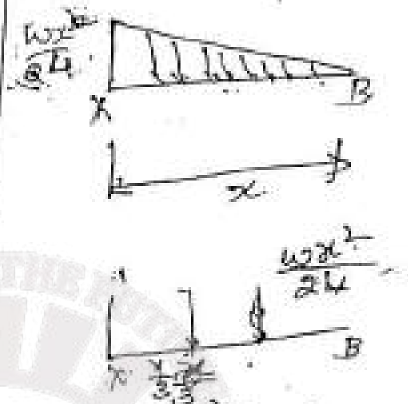
source: diginotes.in

Since UVL acts @ a distance of $\frac{x}{3}$ from max loading side

-ve sign indicates Hogging.

Here the expression represents cubic equation i.e, $\frac{wx^3}{6l}$.

Therefore curve is cubic

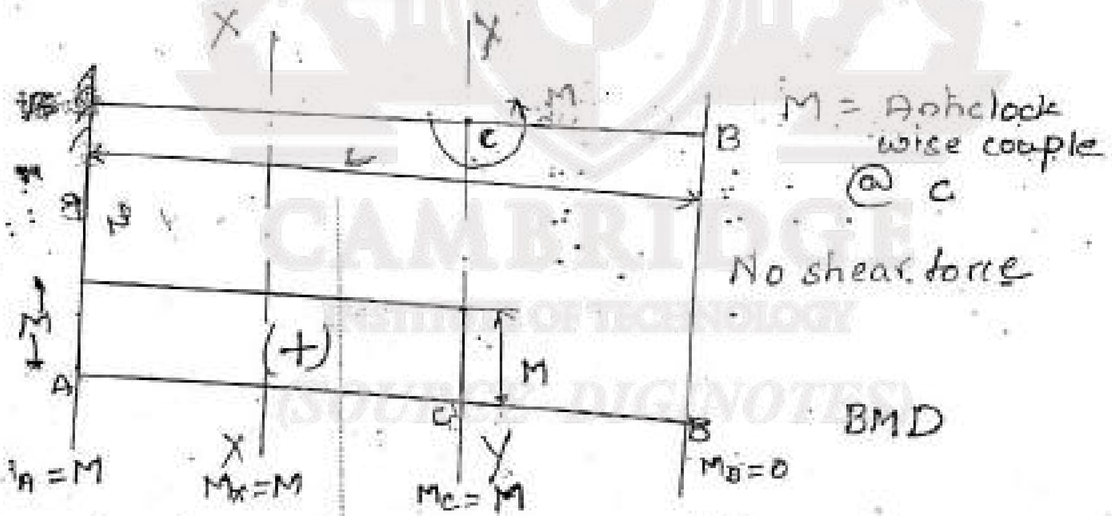


$M_x = -\frac{wx^2}{24} \times \frac{x}{3}$

when $x = 0$, $M_B = 0$

when $x = L$, $M_A = -\frac{wL^2}{6}$

SFD & BMD for a cantilever beam due to Couple @ a section



Only couple is acting on the beam, therefore shear force doesnt exist

Here directly couple is acting @ Y-Y or 'C', therefore there will be no bending moment between C & B.

At every section X-X in between AC, BM = +M.

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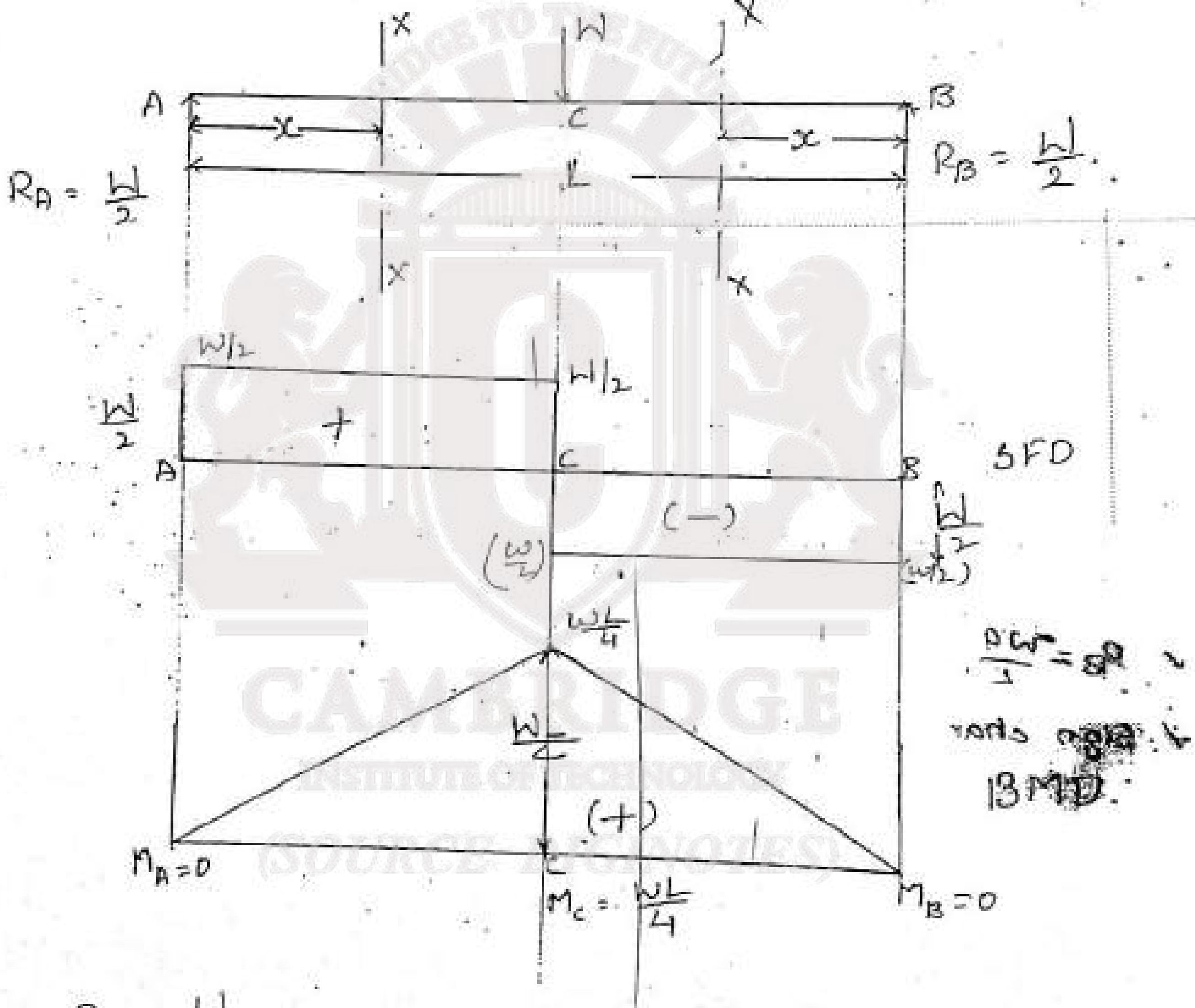
- The B.M. is positive as the moment causes sagging

SIMPLY SUPPORTED BEAM.

SFD & BMD for a simply supported beam with a Point load @ Mid Point.



- ✓ $R_A = R_B = \frac{W}{2}$.
- ✓ $R_A + R_B$ = Total load on beam.
- ✓ SF is constant between A & C [+ve]
- ✓ SF is constant between B & C [−ve]
- ✓ Sign changes @ C, where point load is acting.
- ✓ BM @ A & B = 0
- ✓ BM is maximum @ C
- ✓ $M_C = \frac{WL}{4}$ & +ve
- ✓ BM will be max @ the position of point load acting on a beam

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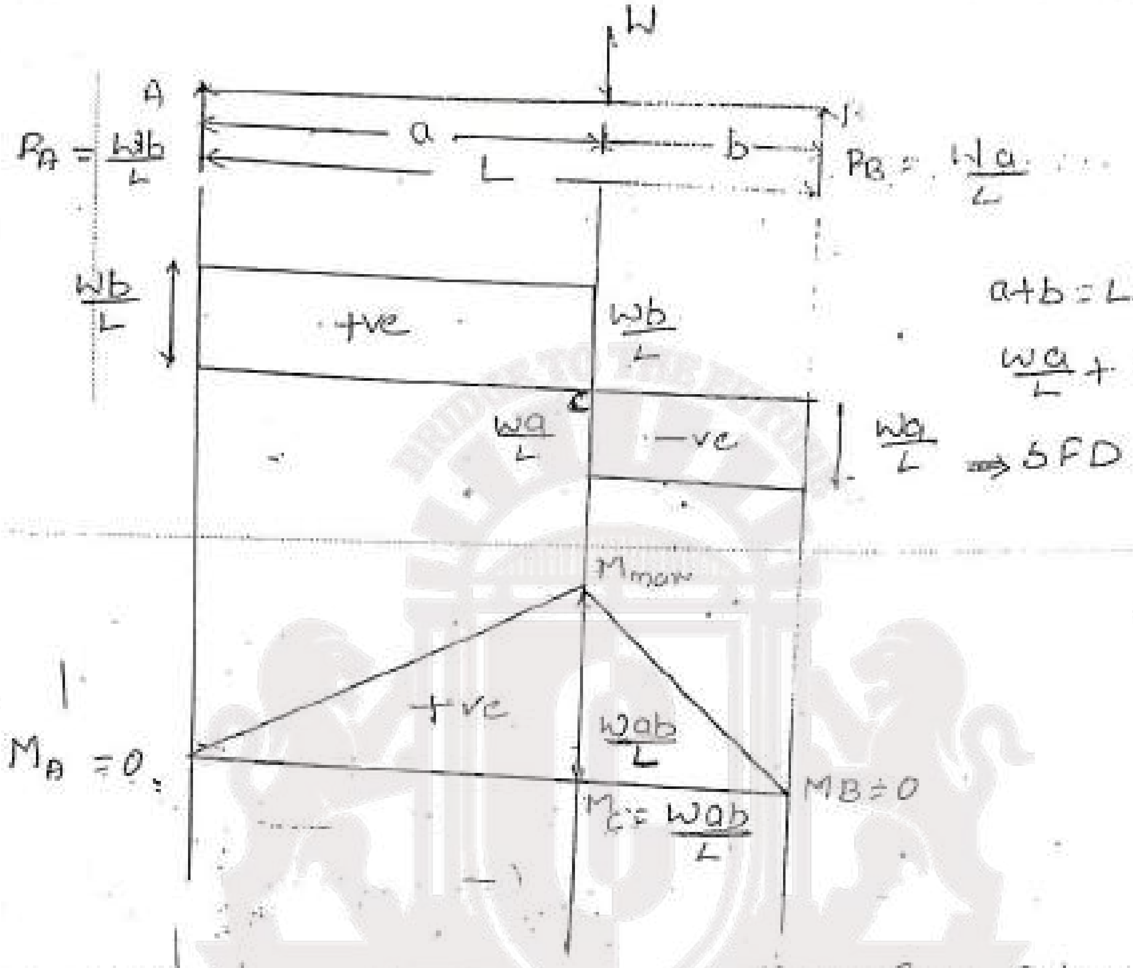
SFD & BMD for a SSB with an eccentric point load.



$a + b = L$

$\frac{Wa}{L} + \frac{Wb}{L} = W$

$\Rightarrow$ SFD

$R_B = \frac{Wa}{L}$, $R_A = \frac{Wb}{L}$

Sign changes @ point load "c".

BMD

Moving from B to A

→ when $x = 0$, $M_B = 0$

→ $x = b$, $M_C = \frac{Wab}{L}$

→ $x = L$, $M_A = 0$

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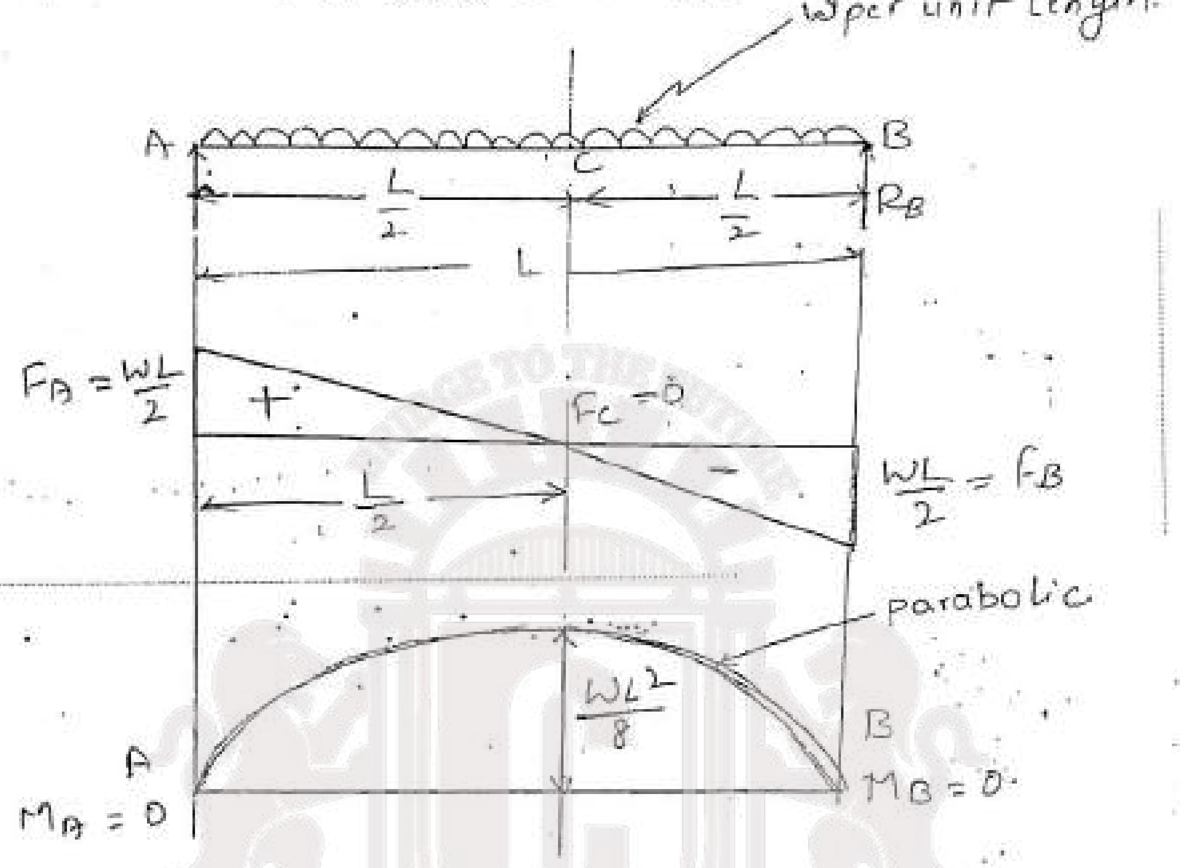
SFD & BMD for a SSB with UDL



✓ SF varies according to the straight line equation

when $x = 0$, $F_B = -\frac{WL}{2}$

when $x = L$, $F_A = \frac{WL}{2}$

when $x = \frac{L}{2}$, $F_C = 0$

✓ BM varies according to the equation of parabola

✓ when $x = 0$, $M_B = 0$

✓ when $x = L$, $M_A = 0$

✓ when $x = L/2$, $M_C = M_{max}$

$M_C = \frac{WL^2}{4} - \frac{WL^2}{8} = \frac{WL^2}{8}$

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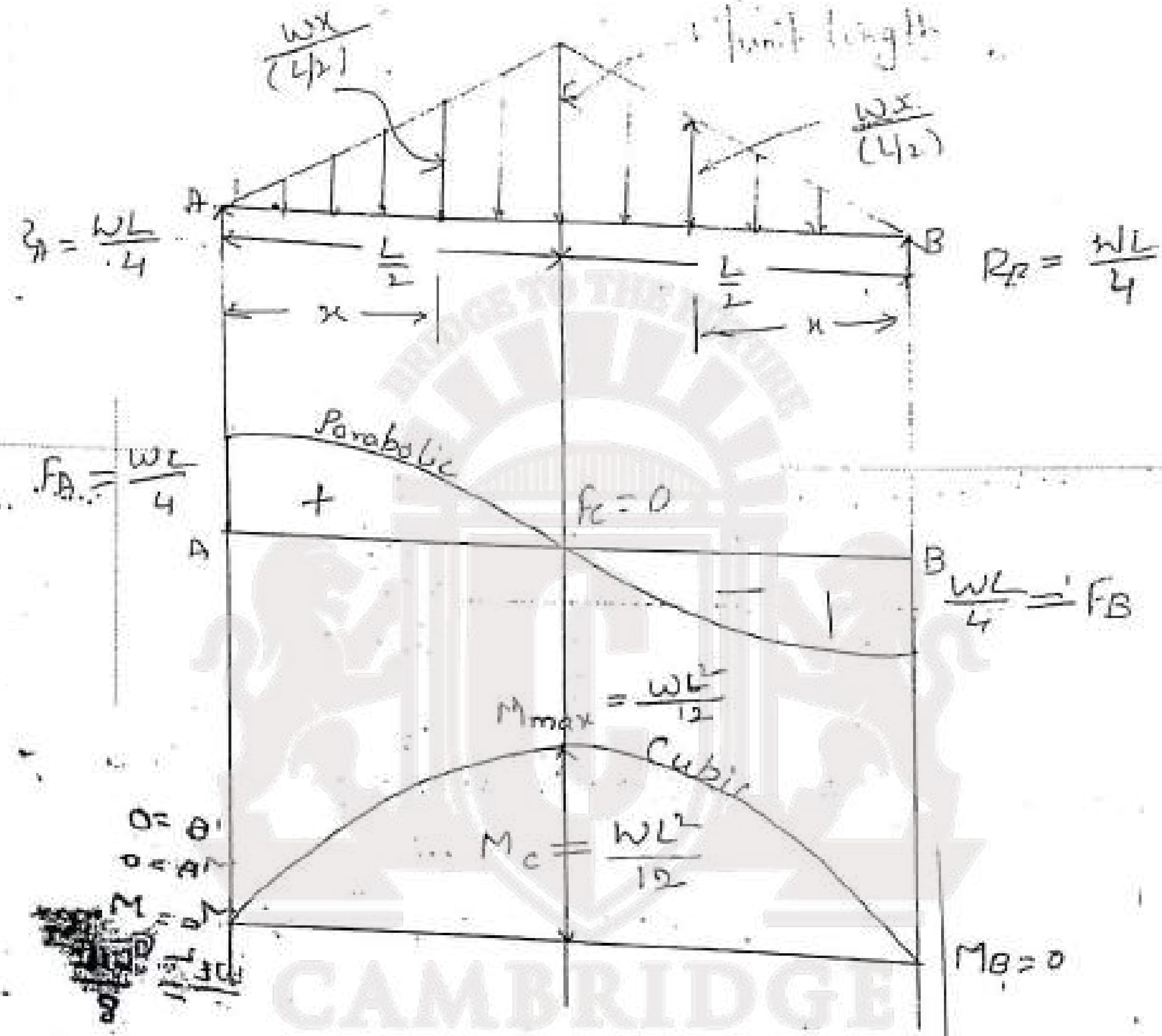
SFD & BMD for a S.S.B. [illegible] load from 'C' @ [illegible] length [illegible]



- SF varies as parabola
- From Right to left

When $x = 0$, $F_B = -\frac{WL}{4}$

$x = \frac{L}{2}$, $F_C = 0$

$x = L$, $F_A = +\frac{WL}{4}$

BM varies according to cubic law

When $x = 0$, $M_B = 0$

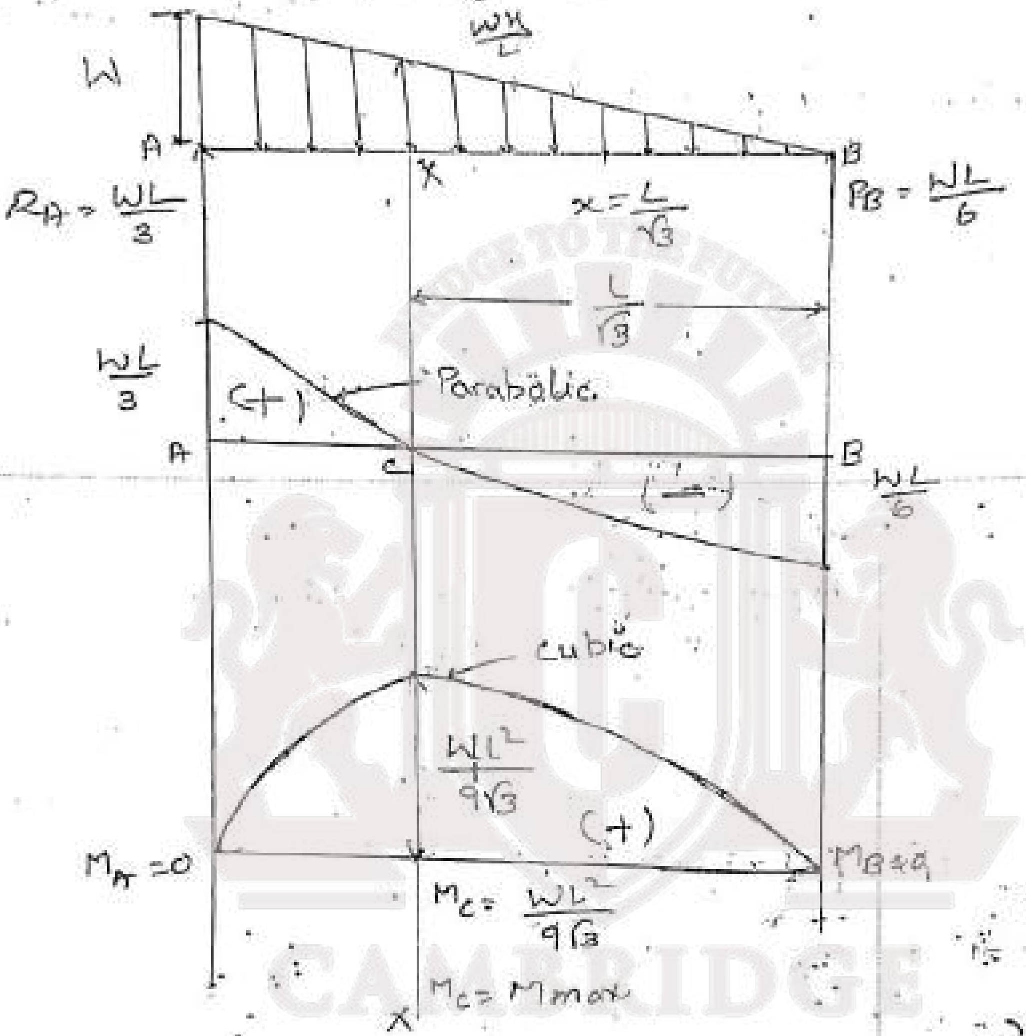
$x = L/2$, $M_C = \frac{WL^2}{12}$

$x = L$, $M_A = 0$

$M = M_{max}$ @ $M = M_C$ or when $x = L/2$

* Maximum Bending Moment occurs @ a point where SF changes its sign i.e @ M_{max} @ C

SFD & BMD for a SSB carrying UVL from 0 @ one end to W/unit length @ the other end



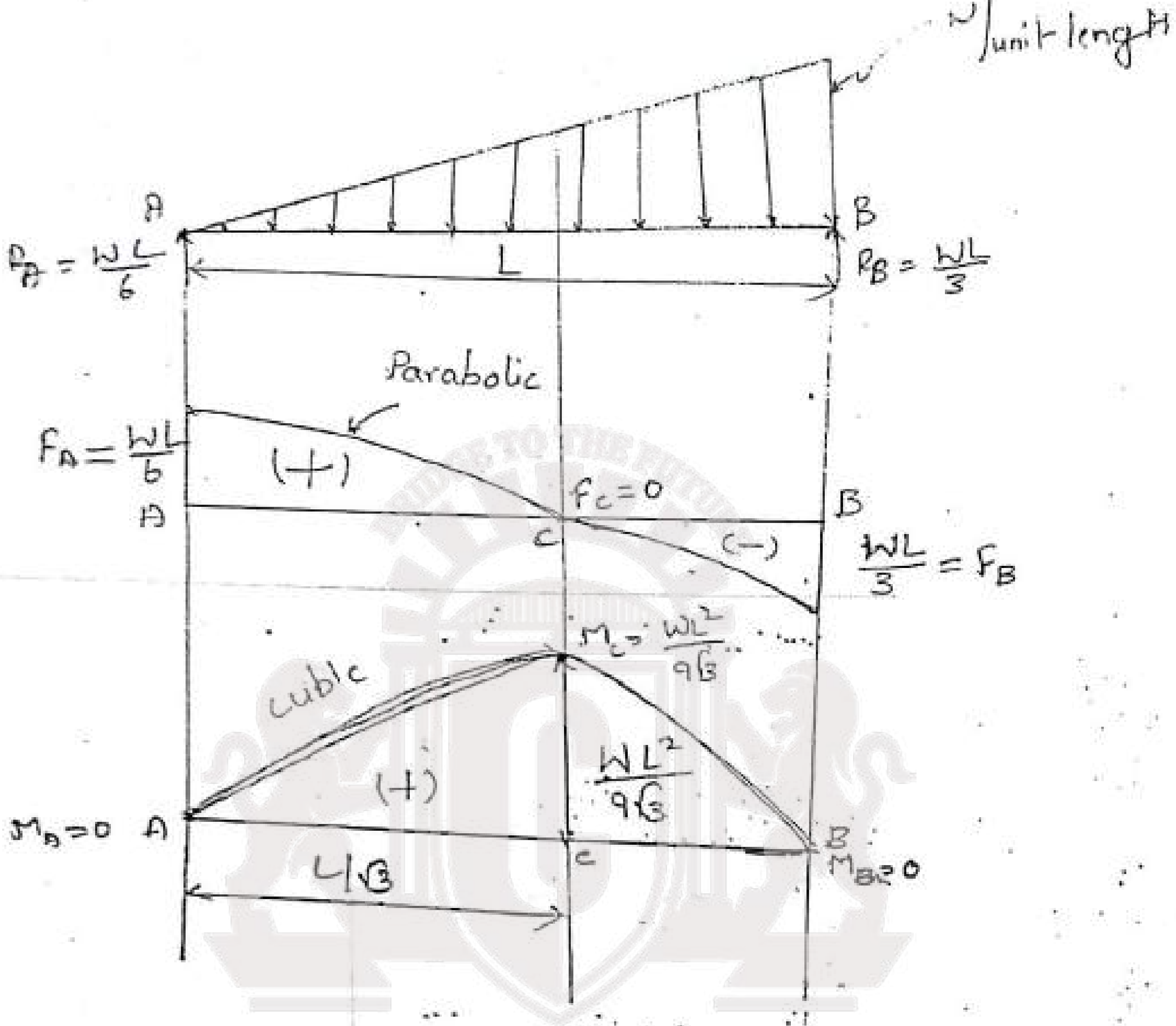
- SF varies according to the equation of parabola.

$SF @ x-x = -R_B + \frac{1}{2} \cdot \frac{Wx}{L} \cdot x = -\frac{WL}{6} + \frac{Wx^2}{2L}$

- BM varies according to the cubic law, $M_{x-x} = \frac{WLx}{6} - \frac{Wx^3}{6L}$
- when $x = 0$, $SF @ B = -\frac{WL}{6}$
  $x = L$, $SF @ A = \frac{WL}{3}$
- SF varies from $-\frac{WL}{6}$ to $\frac{WL}{3}$ &
  $SF = 0 @ C$, distance of x from B
- $x = \frac{L}{\sqrt{3}}$, $SF @ C = 0$

- when $x = 0$, $M_B = 0$
  $x = L$, $M_A = 0$
  $x = \frac{L}{\sqrt{3}}$, $M_C = \frac{WL^2}{9\sqrt{3}}$
- $M_C = M_{max} @ x = \frac{L}{\sqrt{3}}$

source: diginotes.in



$R_A + R_B = \frac{W \times L}{2}$ [$\frac{1}{2}$ x base x height = Load]

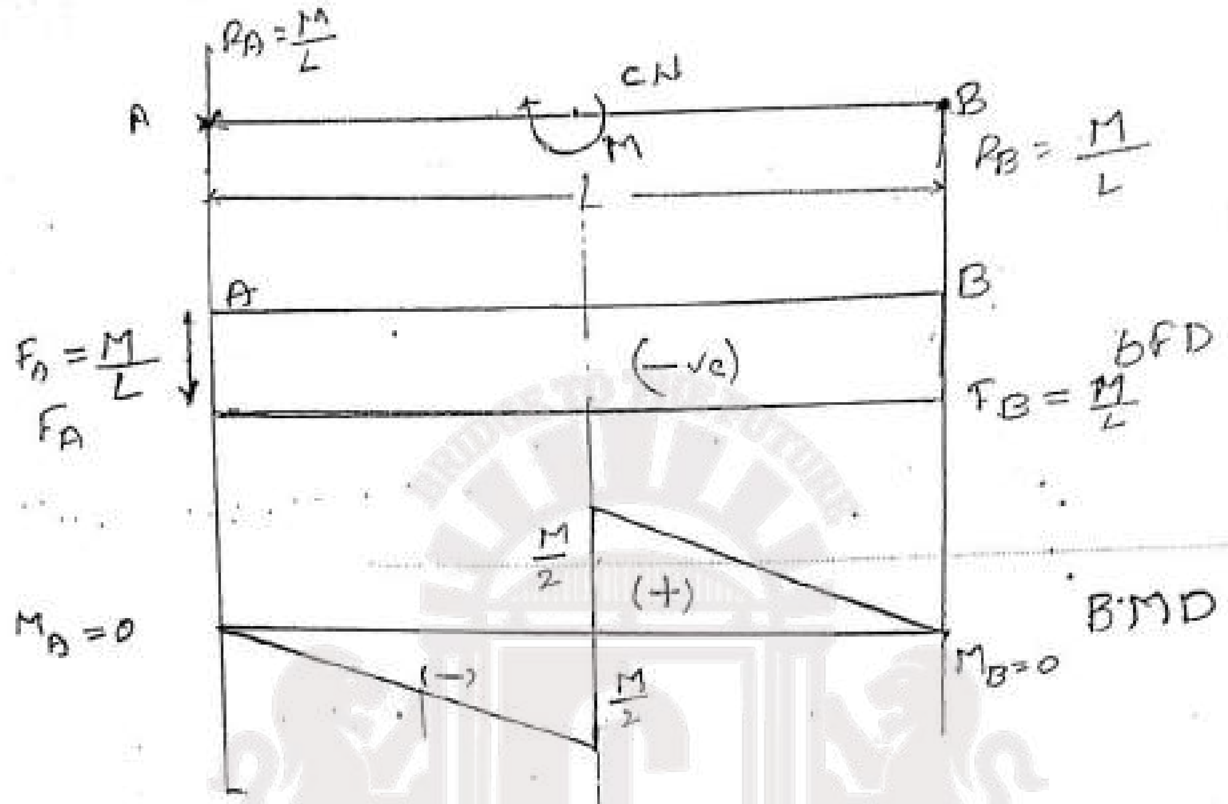
$M_A = 0 \Rightarrow R_B = \frac{WL}{6}$, $R_A = \frac{WL}{3}$

SF @ C = 0

$M_{max} = M_C = \frac{WL^2}{9\sqrt{3}}$ @ $x = \frac{L}{\sqrt{3}}$ from A

source: diginotes.in

SFD & BMD for a SSB carrying couple



Note: Tendency of this couple will be to lift the support @ A & to lower the support @ B. Hence R_A @ A will be in the downward direction & R_B will be upwards.

- $R_A = \frac{M}{L}$ [DOWNWARDS] $\quad M_B = 0$

  $R_B = \frac{M}{L}$ [UPWARDS] $\quad M_A = 0$

- SF is constant @ all sections b/w A & B & $= -\frac{M}{L}$

- When $x = 0$, $F_B = -\frac{M}{L}$ $\quad x = 0$, BM @ B = 0

  $x = L$, $F_A = -\frac{M}{L}$ $\quad x = L$, BM @ A = 0

  $x = L/2$, $C = -\frac{M}{2}$

- At C the BM rises suddenly from $-\frac{M}{2}$ to $+\frac{M}{2}$ due to positive couple

Q. Two shafts of the same material & of same length are subjected to the same T, if the I shaft is of a solid c/s & 2nd shaft is of hollow c/s section, whose internal dia is 2/3 of the outer dia & the max shear stress develop in each shaft is the same, compare the weights of the shafts.

sol:

$$(\tau_{max})_s = (\tau_{max})_h$$

$$\left(\frac{TR}{J}\right)_s = \left(\frac{TR}{J}\right)_h$$

$$T_s = T_h \Rightarrow \left(\frac{R}{J}\right)_s = \left(\frac{R}{J}\right)_h$$

let D_s = dia of solid

D_h = dia of hollow

Given $d_h = \frac{2}{3} D_h$

$$J_s = \frac{\pi}{32} D_s^4 \quad ; \quad J_h = \frac{\pi}{32}\left(D_h^4 - d_h^4\right)$$

$$= \frac{\pi}{32} \times \frac{65}{81} D_h^4$$

$R_s = D_s/2$, $R_h = D_h/2$

from (1) $\Rightarrow$

$$\frac{(D_s/2)}{\frac{\pi}{32}\left(D_s^4\right)} = \frac{(D_h/2)}{\left(\frac{\pi}{32}\right)\left(\frac{65}{81} D_h^4\right)}$$

$$\frac{1}{D_s^3} = \frac{81}{65}\frac{1}{D_h^3}$$

$$\Rightarrow \left(\frac{D_s}{D_h}\right)^3 = \frac{65}{81} \Rightarrow \frac{D_s}{D_h} = 0.9293$$

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Weight W = Volume × Density × Acceleration due to gravity.

$= A \times L \times \rho \times g$

$W_s = \frac{\pi}{4} D_s^2 L \rho g$

$W_h = \frac{\pi}{4} \left[D_h^2 - d_h^2 \right] L \rho g$

$= \frac{\pi}{4} \times \frac{5}{9} D_h^2 L \rho g$

$\therefore \frac{W_s}{W_h} = \frac{\frac{\pi}{4} D_s^2 L \rho g}{\frac{\pi}{4} \times \frac{5}{9} D_h^2 L \rho g} = \frac{9}{5} \left(\frac{D_s}{D_h} \right)^2$

$= \frac{9}{5} (0.9293)^2 = 1.5545$

Q) Define the terms torsional rigidity & torsional strength.

Q. Derive the relation for a circular shaft when subjected to torsion as given below

$\frac{T}{J_P} = \frac{\tau}{R} = \frac{G\theta}{L}$

Q) Find the diameter of the shaft required to transmit 60 kW @ 150 rpm if the max torque is 25% more than the mean torque for a max permissible shear stress of 60 MN/m². Find also the angle of twist for a length of 4 m. Take G = 80 GPa.

source: diginotes.in

Sol: Power depends $P = \frac{2\pi N T_{mean}}{60000}$ kW

$60 = \frac{2\pi \times 150 \times T_{mean}}{60000}$

$T_{mean} = 3819.72$ Nm

$T_{max} = 1.25\, T_{mean}$

$= 1.25 \times 3819.72$

$\boxed{T_{max} = 4774.65 \text{ Nm}}$

$T_{max} = \frac{\pi}{16} D^3 \tau_{max}$

$\tau_{max} = 60 \times 10^6$ N/m²

$4774.65 = \frac{\pi}{16} \times D^3 \times 60 \times 10^6$

$D = 0.074$ m

$D = 74$ mm

$\theta = \frac{TL}{GJ}$

$J = \frac{\pi}{32} D^4 = \frac{\pi}{32} \times 74^4$

$\theta = \frac{4774.65 \times 10^3 \times 4 \times 10^3}{80 \times 10^3 \times \frac{\pi}{32} \times 74^4}$ $= 0.081$ radian

$= 0.081 \times \frac{180}{\pi}$ degree

$\boxed{\theta = 4.64^\circ}$

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source: diginotes.in

Q. A solid shaft rotating @ 500 r.p.m transmits 30 kW. Maximum torque is 20% more than mean torque. Allowable shear stress 65MPa & modulus of rigidity 81 GPa, angle of twist in the shaft should not exceed 1° in 1 meter length. Determine suitable diameter.

Sol: Given data:

$N = 500$ rpm, $P = 30$ kW

$T_{max} = 1.2\ T_{avg}$

$\tau_{allowable} = 65$ MPa

$G = 81$ GPa

$\theta = 1^{\circ}$

$L = 1$m

$$P = \frac{2\pi N T_{avg}}{60}$$

$$30 \times 10^3 = \frac{2 \times \pi \times 500 \times T_{avg}}{60}$$

$$T_{avg} = 572.96 \text{ Nm}$$

$$T_{max} = 1.2\ T_{avg} = 1.2 \times 572.96$$

$$T_{max} = 687.55 \text{ Nm}$$

Torque based on shear stress criterion

$$T_{max} = \frac{\tau_{max}}{R} \times J$$

$$\tau_{max} = 65 \times 10^6 \text{ Pa}$$

$$R = D/2$$

$$J = \pi D^4/32$$

source: diginotes.in

$$T_{max} = \frac{65 \times 10^6}{(D/2)} \times \frac{\pi}{32} (D^4)$$

$$T_{max} = \frac{65 \pi \times 10^6}{16} D^3 \longrightarrow (1)$$

Torque based on angle of twist criterion:

$$\theta = \frac{TL}{GJ}$$

$$\theta_{max} = \frac{T_{max} L}{GJ}$$

Given $\theta_{max} = 1° = \frac{\pi}{180}$ rad

$$T_{max} = \frac{GJ}{L} \theta_{max}$$

$$= \frac{81 \times 10^9 \times \pi \times D^4}{1 \times 32} \times \frac{\pi}{180}$$

$$687.55 = \frac{81 \times 10^9 \times \pi^2 D^4}{640}$$

$D = 0.047$ m $= 47$ mm

$D = 47$ mm

From (1), $687.55 = \frac{65 \times 10^6}{16} \pi D^3$

$D = 0.038 = 38$ mm

As $T_{max} \propto \frac{1}{D^3}$ & $\theta_{max} \propto \frac{1}{D^4}$

larger value of D will satisfy both the condition

$D = 47$ mm

source: diginotes.in

Q) A hollow CI circular section column is 7.5 m long & is pined @ its both ends. The inner diameter of the column is 160 mm & thickness of the wall is 20 mm. Find the safe load by Rankine's formula, using a factor of safety of 5. Also find the slenderness ratio & ratio of Euler & Rankine critical loads. For cast iron take $\sigma_c = 550\ N/mm^2$, $\alpha = \frac{1}{1600}$, $E = 8\times10^4\ N/mm^2$.

Sol: Given data

D = outer diameter = $d + 2t$

= inner diameter + 2 thickness

$= 160 + 2\times20$

$D = 200$ mm

$\Rightarrow A = \frac{\pi}{4}(D^2 - d^2)$ =

$= 11309.73\ mm^2$

FOS = 5

$f_c = \sigma_c = 550\ N/mm^2$

$a = \alpha = \frac{1}{1600}$, $E = 8\times10^4\ N/mm^2$

To calculate

(i) Slenderness Ratio $= \frac{L}{K}$ = $\frac{\text{Effective length}}{\text{Radius of gyration}}$

$K = \sqrt{\frac{I_{min}}{A}}$

source: diginotes.in

$I_{min} = \frac{\pi}{64}(D^4 - d^4) = 4.637 \times 10^7\ mm^4$

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{4.637 \times 10^7}{11309.73}}$

$k = 64.03$ mm

Effective length $L = l$ for pinned @ both ends.

$\therefore S.R = \frac{L}{K} = \frac{7500}{64.03} = 117.133$

ii) Rankin safe load = $\frac{\text{Rankin's Critical load}}{F.O.S}$

$= \frac{P_r}{FOS}$

$P_r = \frac{\sigma_c A}{1+\alpha\left(\frac{L}{k}\right)^2} = \frac{550 \times 11309.73}{1+\left(\frac{1}{1600}\right)(117.133)^2}$

$P_r = 649.64\ kN$

Rankins safe load $= \frac{P_r}{FOS} = \frac{649.64}{5}$

$= 129.928\ KN$

iii) $\frac{\text{Euler Critical load}}{\text{Rankin Critical load}} = \frac{P_E}{P_r}$

$P_E = \frac{\pi^2 EI}{L^2}$ where $L = l$

source: diginotes.in

$$P_E = \frac{\pi^2 \times 8\times10^4 \times 4.637\times10^7}{7500^2}$$

$$P_E = \underline{650.885 \text{ KN}}$$

$$\therefore \frac{P_E}{P_r} = \frac{650.885 \text{ KN}}{649.64 \text{ KN}}$$

$$= \underline{\underline{1.002}}$$

(Q) Design the section of a circular cast iron column that can safely carry a load of 1000 kN. The length of the column is 6m. Take $\alpha = \frac{1}{1600}$, FOS = 3. One end of the column is fixed & other end is free. Critical stress is 560 MPa.

Sol: Given data

$\therefore$ P_{safe} = Safe load = 1000 kN
FOS = 3
Length of column = l = 6m = 6000 mm
$\alpha = \frac{1}{1600}$

Critical stress = σ_c = 560 MPa

One end is fixed & other end is free $\therefore L = 2l$
$L = 12\times10^3$ mm

To calculate: D = ?

$$P_r = \frac{\sigma_c A}{1+\alpha\left(\frac{L}{k}\right)^2}$$

source: diginotes.in

$$A = \frac{\pi D^2}{4}$$

$$k = \sqrt{\frac{I}{A}}$$

$$I = \frac{\pi D^4}{64}$$

$$\Rightarrow k = \sqrt{\frac{\frac{\pi D^4}{64}}{\frac{\pi D^2}{4}}} = \frac{D}{4}$$

$$\therefore P_r = \frac{560 \times \frac{\pi}{4} \times D^2}{1 + \frac{1}{1600}\left(\frac{12 \times 10^3}{D/4}\right)^2}$$

$$3 \times 10^6 = \frac{140 \pi D^2}{1 + \frac{1.44 \times 10^6}{D^2}} = \frac{140 \pi D^2}{\left(\frac{D^2 + 1.44 \times 10^6}{D^2}\right)}$$

$$3 \times 10^6 (D^2 + 1.44 \times 10^6) = 140 \pi D^4$$

$$\Rightarrow 140 \pi D^4 - (3 \times 10^6) D^2 - (3 \times 10^6 \times 1.44 \times 10^6) = 0$$

Take $D^2 = x \Rightarrow D^4 = x^2$

$$\Rightarrow (140 \pi) x^2 - (3 \times 10^6) x - (3 \times 1.44 \times 10^{12}) = 0$$

$$\Rightarrow x = D^2 = 102575.f$$

from $x = \frac{+(3 \times 10^6) \pm \sqrt{(3 \times 10^6)^2 + 4 \times 140 \times \pi \times 3 \times 1.44 \times 10^{12}}}{2 \times 140 \times \pi}$

source: diginotes.in

$x = D^2 = 102575.8 \text{ mm}^2$

$D = 320.27 \text{ mm}$

Q) With a neat what are the assumptions & limitations of Euler's formula.

Q) Derive an Expression for Euler's buckling load for a long column having one end fixed & other end is free. State the assumptions made in the derivation.

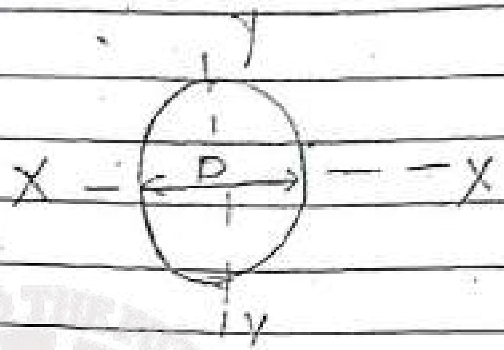
Q) what is meant by effective length of a column? state the value of effective length for various column end conditions.

Q) Explain how the deflection in beam can be reduced.

source: diginotes.in

Formulas used.

For circular Solid shaft.



$\Rightarrow A = \frac{\pi D^2}{4}, m^2$

$\Rightarrow I_{xx} = I_{yy} = \frac{\pi D^4}{64}, m^4$

$I_{zz} = I_{xx} + I_{yy} = J$

$D = 2R$

$\Rightarrow J = \frac{\pi D^4}{32}, m^4$

$\Rightarrow$ Sectional Modulus $= Z = \frac{J}{R} = \frac{2J}{D}$

$Z = \frac{\pi D^3}{16}, m^3$

For circular Hollow shaft.

D = outer diameter m

d = Inner diameter m

$d = 2r$

$D = 2R$

Area of hollow shaft = A_h

$A_h = \frac{\pi}{4}(D^2 - d^2)\ m^2$

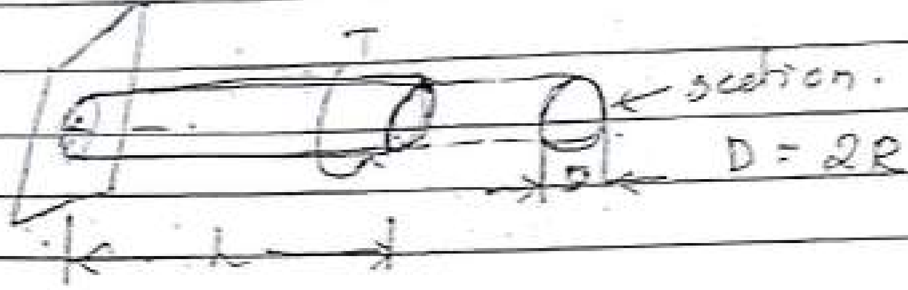
$I_{xx} = \frac{\pi}{64}(D^4 - d^4), m^4 \Rightarrow I_{yy} = I_{xx}$.

$J = I_{xx} + I_{yy} = \frac{\pi}{32}(D^4 - d^4), m^4$

$Z = J/R = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right), m^3$

source: diginotes.in

Torsional Equation.



Let

T = Torque in N-m

L = length of shaft in m

D = Diameter " " in m

R = Radius of " in m

θ = Angle of Twist in rad.

G = Modulus of Rigidity in N/m^2

τ = Shear Stress in N/m^2

J = Polar Moment of Inertia in m^4

I_{xx}, I_{yy} = Moment of Inertia in x, y & Z axis in m^4

Z = Section Modulus in m^3

A = Area of shaft in m^2

ϕ = Shear strain, no unit

Formula:

$$I_{xx} = \int r^2 dA = I_{yy}$$

$$J = I_{xx} + I_{yy}$$

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}}$$

$T = F \times R$ $\qquad G = \tau/\phi$

$\tau = F/Area$ $\qquad F = ma$ $\qquad T = I\alpha$

source: diginotes.in

$$\text{Power} = P = \frac{2\pi NT}{60 \times 1000} \text{ kW}$$

where

N = no. of revolutions/min in rpm

T = mean torque or torque of shaft.

$\omega = 2\pi N$

$P = \omega T$

Weight = mass × g = ρ × v × g = ALρg

mass = volume × density = ρV

volume = Area × Length

Torsional stiffness: The torque per unit angle of twist

$$\frac{T}{\theta} = \frac{GJ}{L}, \quad \frac{T}{1} = k \quad \text{when } \theta \,\&\, L = 1$$

Torsional Rigidity: The torque per unit angle of twist for unit length

$$\frac{T}{\theta} L = GJ$$

$$K = GJ, \quad \text{when } \theta = 1, L = 1$$

Torsional strength: The torque per maximum shear stress

$$\frac{T}{\tau_{max}} = \frac{J}{R} = Z$$

Torsional strength = Polar sectional modular Z

source: diginotes.in

Note: If any 2 shafts have equal weight, mass & length and are of same material, then equate their weights & we get

$$(D^2 - d^2)_h = D_s^2$$

where D_h, d_h = Outer & Inner dia of Hollow shaft
D_s = Diameter of solid shaft

For given hollow & solid circular shafts, equate their weight & we get above equation or relationship between diameter of both shafts.

If they asked to prove hollow shaft is stronger than solid shaft, then prove.

$$\frac{T_h}{T_s} > 1 \quad , \quad T_h > T_s$$

If they asked to prove H.S is stiffer than S.S then prove: $\frac{K_h}{K_s} > 1$, $K_h > K_s$

where K_h, K_s = stiffnesses of H.S & S.S

$$K_h = (GJ)_h \quad , \quad K_s = (GJ)_s$$

Weight of solid shaft is greater that hollow shaft.

$$W_s > W_h$$

where, W_s = Weight of solid shaft (S.S)
W_h = Weight of hollow shaft (H.S)

source: diginotes.in

Percentage saving in weight = $\frac{W_s - W_h}{W_s} \times 100$

Percentage saving in Area = $\frac{A_s - A_h}{A_s} \times 100$

assuming W_s & A_s are greater than W_h & A_h

∴

Shear stress always depends on radius of shaft.

if $r = 0$, $\tau = 0$

$r = r_{max}$, $\tau = \tau_{max}$ &

r, τ always varies linearly

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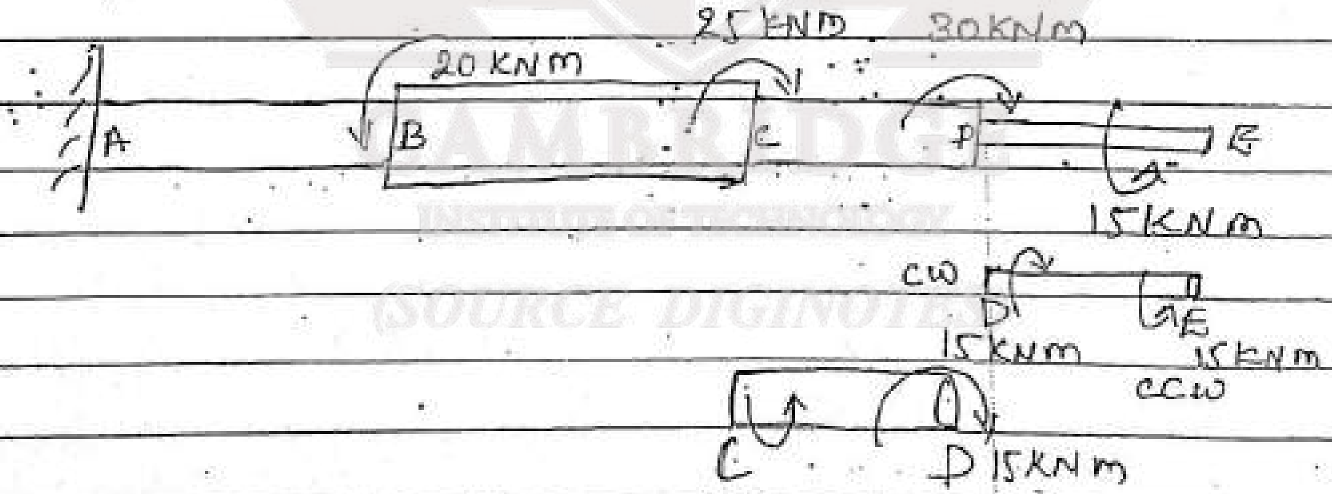
STEPPED SHAFTS

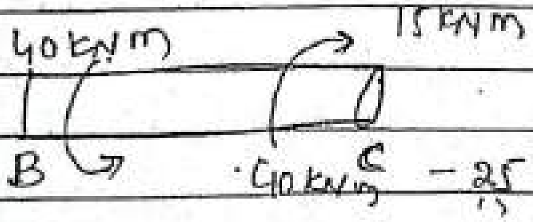
2 or more shafts are combined, having same material & different cross-sections (or) same c/s & different materials.

To find "Torque resisted" by each portion, the following points are to be noted.

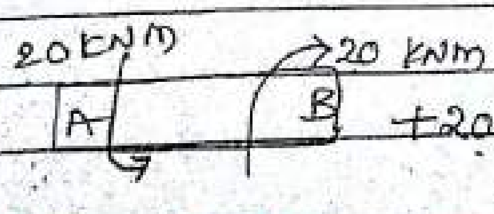
- Torque developed @ the ends of any portion are equal & opposite.
- At common point between two portion, "angle of twist" is the same.
- Torque acting on each portion is obtained from equilibrium condition & calculated same as stepped bars.



15 KNm $-30+15=-15$ KNm



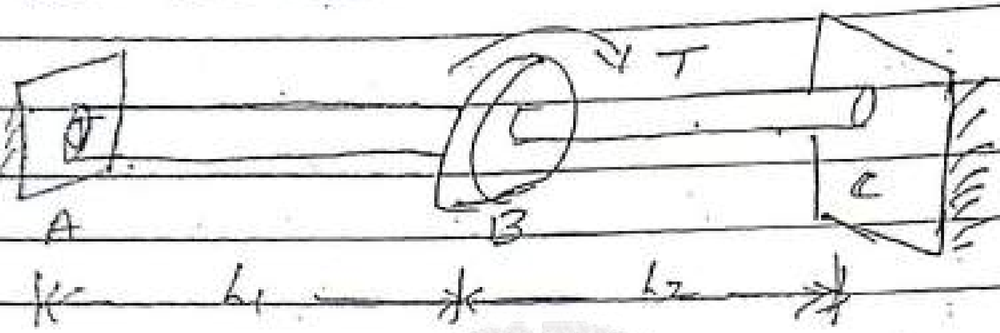
$-25-30+15=-40$ KNm



$+20-25-30+15=-20$ KNm

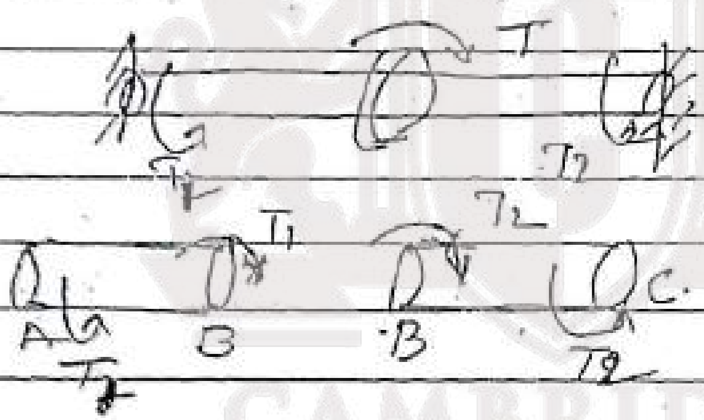
source: diginotes.in

Both ends are fixed.



The above shaft is fixed @ both ends & is subjected to torque T @ the common point

let T_1 & T_2 are torques developed @ ends.



Then $T = T_1 + T_2$

& $\theta_{1B} = \theta_{2B}$

$$\frac{T_1 L_1}{G_1 J_1} = \frac{T_2 L_2}{G_2 J_2}$$

Using above equations, unknown parameter can be calculated.

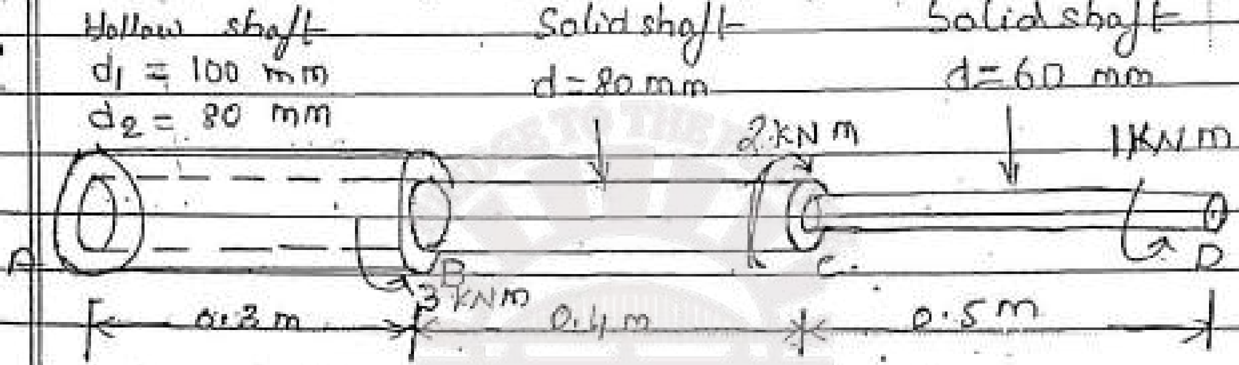
The above concept is applicable for compound bars or shafts as well as.

source: diginotes.in

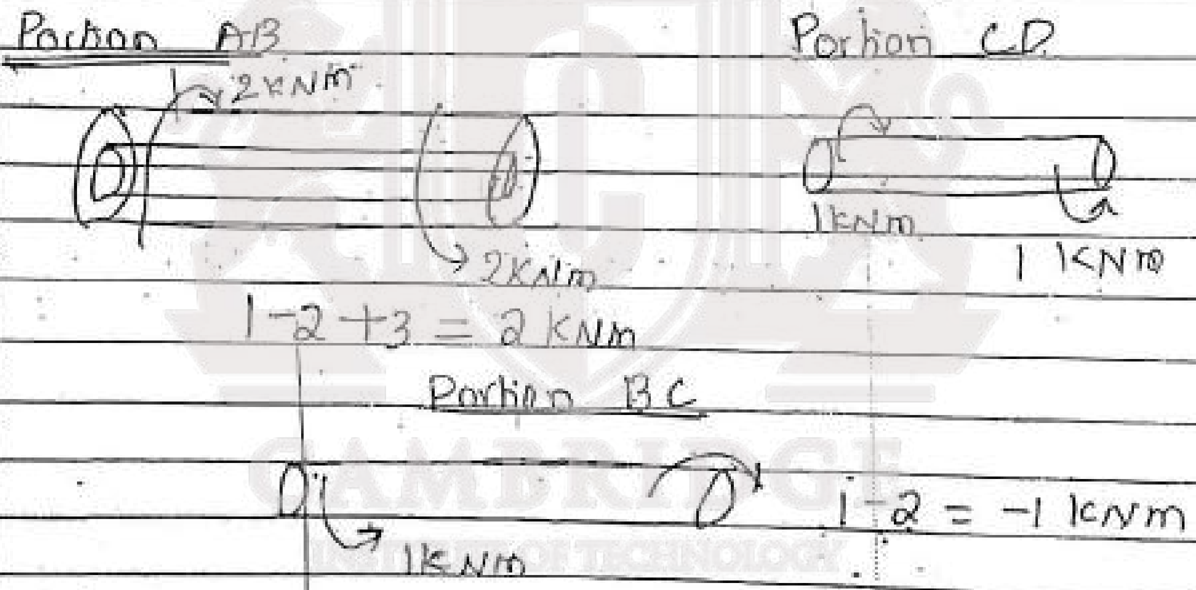
Problems on stepped & Compound shafts.

Q. A stepped shaft is subjected to torque as shown below. Determine angle of twist @ free end and maximum shear stress @ any step. Take $G = 80\ kN/mm^2$.

Sol'



Sol'



Polar modulus of sections

AB ⇒ $J_1 = \frac{\pi}{32}(d_1^4 - d_2^4)$

$= \frac{\pi}{32}(100^4 - 80^4) = 5.8 \times 10^6\ mm^4$

BC ⇒ $J_2 = \frac{\pi D^4}{32} = \frac{\pi \times 80^4}{32} = 4.02 \times 10^6\ mm^4$

source: diginotes.in

CD $\Rightarrow J_3 = \frac{\pi D^4}{32} = \frac{\pi}{32} \times 60^4 = 1.3 \times 10^6 \text{ mm}^4$

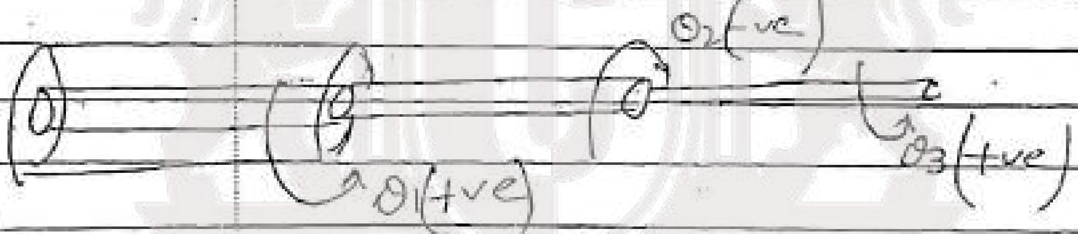
Angle of Twist $\theta = \frac{TL}{GJ}$

$\theta_1 = \frac{T_1 L_1}{G_1 J_1}$, $\theta_2 = \frac{T_2 L_2}{G_2 J_2}$, $\theta_3 = \frac{T_3 L_3}{G_3 J_3}$

Angle of twist @ free end = Angle of twist of AB − Angle of twist of BC + Angle of twist of CD.

$= \theta_1 - \theta_2 + \theta_3$

Note:



$\therefore$ Angle of twist @ free end $= \frac{1}{G}\left(\frac{T_1 L_1}{J_1} - \frac{T_2 L_2}{J_2} + \frac{T_3 L_3}{J_3}\right)$

$= \frac{1}{80 \times 10^3}\left[\frac{2 \times 10^3 \times 10^3 \times 300}{5.8 \times 10^6} - \frac{1 \times 10^6 \times 400}{4.02 \times 10^6} + \frac{1 \times 10^6 \times 400}{1.3 \times 10^6}\right]$

$= 4.97 \times 10^{-3}$ rad.

Shear stresses

AB, $\left(\frac{T}{J}\right)_1 = \left(\frac{\tau}{R}\right)_1 \Rightarrow \tau_1 = \left(\frac{TR}{J}\right)_1$

source: diginotes.in

$$\tau_1 = \frac{2\times10^6 \times 50}{5.8\times10^6} = 17.25\ N/mm^2$$

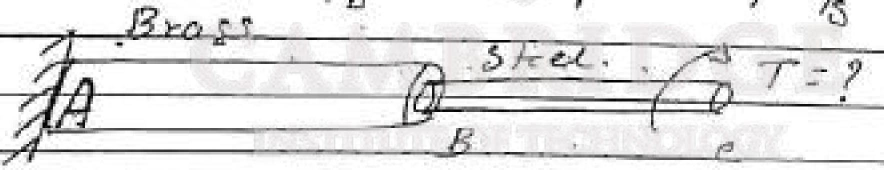
$$BC,\ \tau_2 = \frac{T_2 R_2}{J_2} = \frac{1\times10^6\times 40}{4.02\times10^6} = 9.95\ N/mm^2$$

$$CD,\ \tau_3 = \frac{T_3 R_3}{J_3} = \frac{1\times10^6\times 30}{1.28\times10^6} = 23.58\ N/mm^2$$

From above 3 torsion values, τ will be maximum in portion CD.

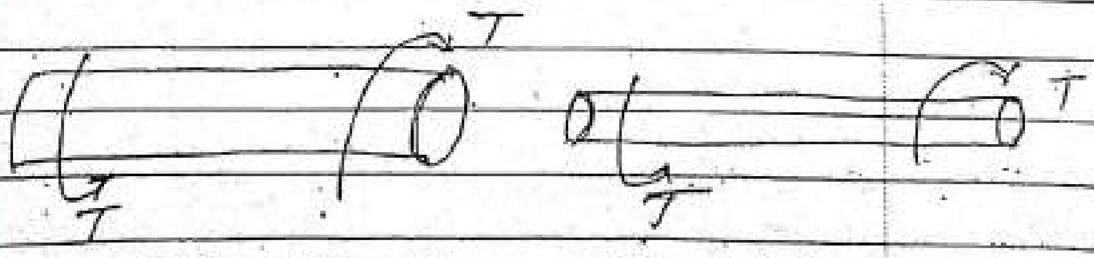
$\therefore\ \tau_{max} = 23.58\ N/mm^2$

(2) The allowable shear stress in brass is 80 N/mm² & in steel 100 N/mm². Find the maximum torque that can be applied in the stepped shaft shown below. Also find the total rotation of free end w.r.t the fixed end. Take $G_b = 40\ kN/mm^2$ & $G_s = 80\ kN/mm^2$



$D_B = 80$ mm. $D_S = 60$ mm.

$L_B = 1$m $L_S = 1.2$ m.



Both shaft portions subjected to same torque T

Brass Rod AB: $\tau_B = 80\ N/mm^2$

$R = 40\ mm$

$J_B = \frac{\pi D_B^4}{32} = \frac{\pi \times 80^4}{32}$

$\Rightarrow \frac{T_B}{J_B} = \frac{\tau_B}{R_B}$

$\Rightarrow T_B = \frac{\tau_B J_B}{R_B} = \underline{8042477\ N\text{-}mm}$

Steel Rod BC: $\tau_c = 100\ N/mm^2$

$R_c = 30\ mm$

$J_c = \frac{\pi \times D_c^4}{32} = \frac{\pi \times 60^4}{32}$

$\Rightarrow T_c = \frac{\tau_c J_c}{R_c} = \underline{4241150.1\ N\text{-}mm}$

∴ Max Torque that can be applied = 4241150.1 N-m

Rotation of free end $= \theta_b + \theta_s$

$= \frac{T L_b}{G_b J_b} + \frac{T L_c}{G_c J_c}$

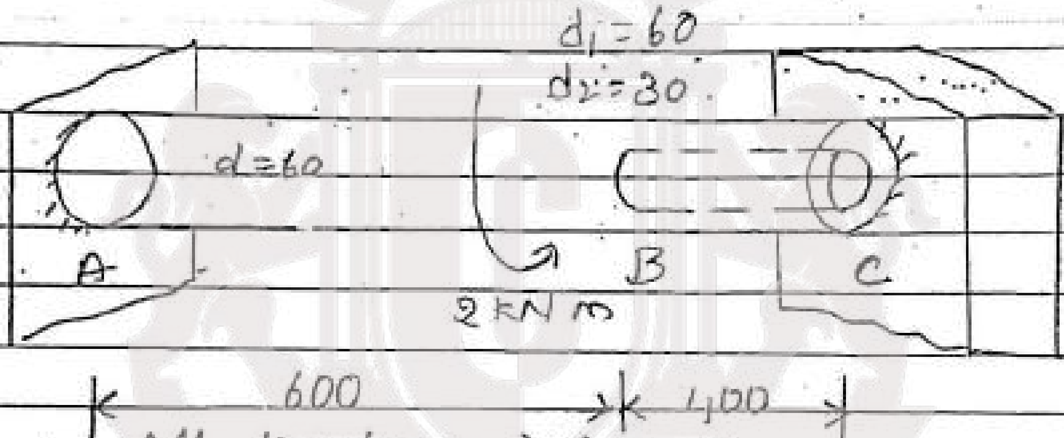
$= 4241150.1 \left(\frac{1000}{4\times10^3 \times \frac{\pi \times 80^4}{32}} + \frac{1200}{80\times10^3 \times \frac{\pi \times 60^4}{32}} \right)$

$= \underline{0.0764\ rad}$

source: diginotes.in

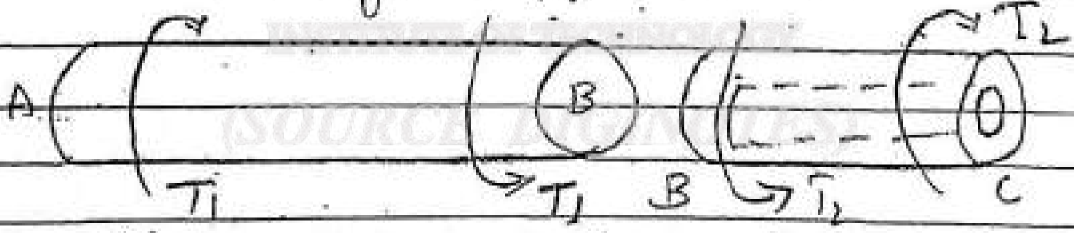
Problem based on Both Ends are fixed.

Q) A bar of length 1000 mm & diameter 60 mm is centrally bored for 400 mm, the bore diameter being 30 mm as shown below. If the 2 ends are fixed & is subjected to a torque of 2KN-m as shown below, find the maximum stresses developed in the two portions.



All dimensions in "mm"

Sol. Free body diagram



Let T_1 = Torque by portion AB

T_2 = Torque on portion BC.

θ_1 = Twist in portion AB @ B

θ_2 = " " " BC @ B

T = Total Torque

$T = T_1 + T_2 = 2 KN m$

source: diginotes.in

$$\theta_1 = \frac{T_1 L_1}{G_1 J_1} = \frac{T_1 \times 600}{G \times \frac{\pi}{32} \times 60^4}$$

$$\theta_2 = \frac{T_2 L_2}{G_2 J_2} = \frac{T_2}{G} \times \frac{400 \times 32}{\pi \times (60^4 - 30^4)}$$

Since $\theta_1 = \theta_2$ for consistency of deformation,

$G_1 = G_2$

$$\frac{600}{60^4} T_1 = \frac{400 \, T_2}{60^4(1 - 0.5^4)} \qquad [\because 30/60 = 0.5$$

$$T_1 = 0.7111 \, T_2$$

& we know that $T = T_1 + T_2$

$$2 \times 10^6 = 0.7111 \, T_2 + T_2$$

$$T_2 = 1.1688 \times 10^6 \text{ N-mm}$$

$$T_1 = 0.8311 \times 10^6 \text{ N-mm}$$

To find maximum stress in portion AB & BC

(i) Portion AB

$$\frac{T_1}{J_1} = \frac{\tau_1}{R_1} \Rightarrow \tau_1 = \frac{T_1 \cdot R_1}{J_1}$$

$$\tau_1 = \frac{0.8311 \times 10^6 \times 30}{\frac{\pi}{32} \times 60^4} = \underline{19.60 \text{ N/mm}^2}$$

(ii) Portion BC

$$\tau_2 = \frac{T_2 R_2}{J_2} = \frac{1.1688 \times 10^6 \times 30}{\left(\frac{\pi}{32}\right)(60^4 - 30^4)}$$

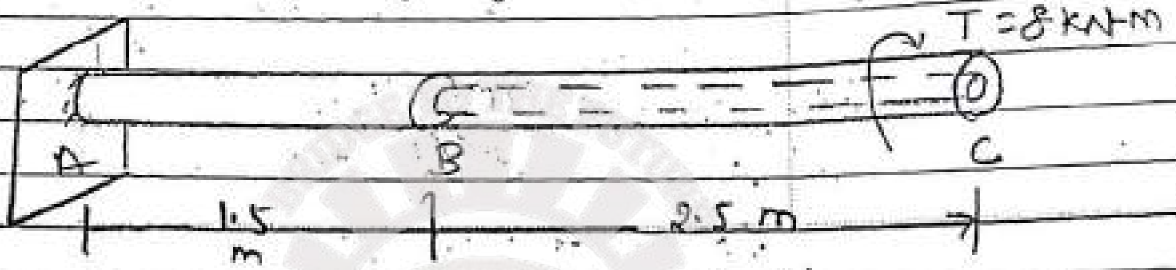
$$\boxed{\tau_2 = 29.39 \text{ N/mm}^2}$$

source: diginotes.in

Problem on Same material different c/s:-

Q. The shaft shown below is securely fixed @ A & is subjected to a torque of 8 kN-m. If portion AB is solid shaft of 100 mm diameter & portion BC is hollow with external diameter 100 mm & internal diameter 75 mm. Find the max stress & max angle of twist. Take G = 80 kN/mm².

Sol:



Sol: FBD

Each portion is subjected to torque T = 8 kN-m.

$T = 8 \times 10^6$ N-mm

$G = 80 \times 10^3$ N/mm²

$L_1 = 1.5$ m $= 1.5 \times 10^3$ mm

$L_2 = 2.5$ m $= 2.5 \times 10^3$ mm

Hollow shaft $\begin{cases} D_1 = 100 \text{ mm} \\ D_2 = 75 \text{ mm} \end{cases}$

Solid shaft $D = 100$ mm

θ_1 & θ_2 = Angle of twist in portion AB & BC.

Portion AB

$$J_1 = \frac{\pi D^4}{32} = \frac{\pi \times 100^4}{32} = 9.82 \times 10^6 \text{ mm}^4$$

source: diginotes.in

$$\frac{T_1}{J_1} = \frac{\tau_1}{R_1}$$

$$T_1 = \tau_1 \times Z_1 \qquad \left(J_1/R_1 = Z_1 \right)$$

$$T = \tau_1 \times \frac{\pi D^3}{16} \rightarrow (1)$$

Portion BC

$$J_2 = \frac{\pi (D_1^4 - D_2^4)}{32} = 6.72 \times 10^6 \ mm^4$$

$$T = \tau_2 \times \frac{J_2}{R_2} = \tau_2 (Z_2) \rightarrow (2)$$

From (1) & (2)

Since T is same on portion AB & BC, & $Z_2 > Z_1$ since $J_1/R_1 < J_2/R_2$

$\therefore \tau_2 = \tau_{max}$ in case of portion BC & it is calculated as

$$T = \tau_2 \times \frac{6.72 \times 10^6}{50}$$

$$\Rightarrow \frac{8 \times 10^6 \times 50}{6.72 \times 10^6} = \tau_2 = \tau_{max}$$

$$\Rightarrow \boxed{59.60 \ N/mm^2 = \tau_{max}}$$

Note: It can be compared by calculating both τ values i.e. τ_1 & τ_2, after calculating which ever is maximum, take it as τ_{max}.

source: diginotes.in

$\Rightarrow$ Total rotation @ c = $\theta_1 + \theta_2 = \frac{TL_1}{GJ_1} + \frac{TL_2}{GJ_2}$

$$= \frac{8\times10^6}{80\times10^3}\left[\frac{1500}{9.82\times10^6} + \frac{2500}{9.72\times10^6}\right]$$

$\theta = 0.05253$ radians.

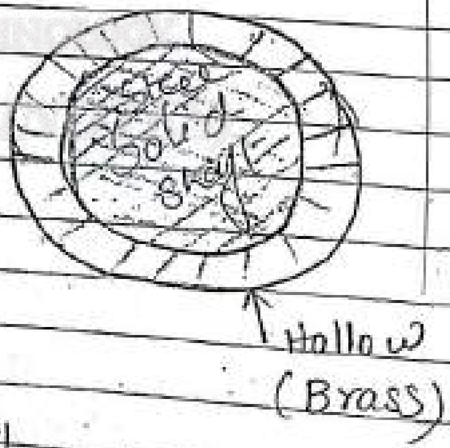
PROBLEMS ON COMPOUND SHAFTS

Q. A brass tube of external diameter 80 mm & internal diameter 50 mm is closely fitted to a steel rod of 50 mm diameter to form a composite shaft. If a torque of 6 KN-m is to be resisted by this shaft, find the maximum stresses developed in each material & the angle of twist in 2m length.
Take $G_B = 40\times10^3$ N/mm²

$G_S = 80\times10^3$ N/mm²

Sol: Given data.

Brass Steel



$d_B = 50$ mm $D_S = 50$ mm

$D_B = 80$ mm

$T_B = ?$ $T_S = ?$

$T = T_S + T_B = 6$ KN-m.

$\tau_B = ?$ $\tau_S = ?$

$\theta_B = ?$ $\theta_S = ?$

$G_B = 40\times10^3$ N/mm² $G_S = 80\times10^3$ N/mm²

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source: diginotes.in

we know that, for a compound shaft,

$\theta_S = \theta_B$ —— ①

$T = T_S + T_B$ —— ②

$$J_S = \frac{\pi D_S^4}{32} = \frac{\pi \times 50^4}{32}$$

$$= 613592.32 \ mm^4$$

$$J_B = \frac{\pi}{32}\left(D_h^4 - d_h^4\right)$$

$$= \frac{\pi}{32}\left(80^4 - 50^4\right) = 3407646.3 \ mm^4$$

$\Rightarrow \theta_S = \theta_B$

$$\frac{T_S L_S}{G_S J_S} = \frac{T_B L_B}{G_B J_B}$$

$$\Rightarrow T_S = \frac{G_S}{G_B} \times \frac{J_S}{J_B} \times T_B \qquad \therefore L_S = L_B$$

$$= \frac{80 \times 10^3}{40 \times 10^3} \times \frac{613592.32}{3407646.3} T_B$$

$$\boxed{T_S = 0.360 \ T_B} \rightarrow \text{SUB in ②}$$

$T = T_S + T_B$

$6 \times 10^6 = 0.36 \ T_B + T_B$

$T_B = 4.411 \times 10^6 \ N\text{-}mm$

$T_S = 1.588 \times 10^6 \ N\text{-}mm$

source: diginotes.in

$\therefore$ Angle of twist $= \theta = \theta_S = \theta_B$

$$\theta = \frac{T_S L_S}{G_S J_S} = 0.0647 \text{ radians}$$

⇒ Maximum shear stress in steel:

$$\frac{T_S}{J_S} = \frac{\tau_{S\,max}}{R_S} \Rightarrow \tau_{S\,max} = \frac{T_S \times R_S}{J_S}$$

$$\Rightarrow \tau_{S\,max} = \frac{1.589 \times 10^6 \times 25}{613592.32}$$

$$\tau_{S\,max} = 64.701 \text{ N/mm}^2$$

⇒ Maximum shear stress in Brass:

$$\frac{T_B}{J_B} = \frac{\tau_{B\,max}}{R_{B\,max}} \Rightarrow \tau_{B\,max} = \frac{T_B R_B}{J_B}$$

$$\Rightarrow \tau_{B\,max} = \frac{4.411 \times 10^6 \times 40}{3407646.3}$$

$$\tau_{B\,max} = 51.778 \text{ N/mm}^2$$

Q) A composite shaft has an aluminium tube of external diameter 60mm & internal dia 40 mm closely fitted to a steel rod of 40mm. If the permissible stress is 60 N/mm² in aluminium & 100 N/mm² in steel, find the maximum torque

source: diginotes.in

the composite section can take. Given $G_a = 27\ kN/mm^2$, $G_s = 80\ kN/mm^2$

Sol: Given data

Aluminium

$D_A = 60\ mm$

$d_A = 40\ mm$

$\tau_A = 60\ N/mm^2$

$G_A = 27\ kN/mm^2$

$T_A = ?$

Steel.

$D_s = 40\ mm$

$\tau_s = 100\ N/mm^2$

$G_s = 80\ kN/mm^2$

$T_s = ?$

$$T = T_A + T_s \rightarrow (1)$$

$J_A = \frac{\pi}{32}(D_A^4 - d_A^4)$ $\quad J_s = \frac{\pi D^4}{32}$ mm^4

$$\theta_s = \theta_A = \theta \rightarrow (2)$$

$J_A = \frac{\pi}{32}(60^4 - 40^4) = 1.02$ mm^4

$J_s = \frac{\pi}{32}(40^4) = 251327.4$ mm^4

From (2), $\theta_s = \theta_A$

$\frac{T_s L_s}{J_s G_s} = \frac{T_A \cdot L_A}{G_A J_A}$ & $L_A = L_s$.

$$\Rightarrow T_s = \frac{G_s}{G_A} \times \frac{J_s}{J_A} \times T_A$$

$$T_s = 0.7293\ T_a \rightarrow (3)$$

From $\left(\frac{T}{J}\right)_s = \left(\frac{\tau}{R}\right)_s$

$T_s = \tau_s \times J_s / R_s$

source: diginotes.in

$T_s = 1.2566$ KN-m

$\Rightarrow T_a = \frac{T_s}{0.7293} = 1.723 \times 10^6$ N-mm

$T = T_s + T_a = \underline{2.978}$ KN-m → (4)

From $\left(\frac{T}{J}\right)_A = \left(\frac{\tau}{R}\right)_A$

$\Rightarrow T_A = 2.042$ KN-m

$T_S = T_A \times 0.7293$

$T_S = 1.4893$ KN-m

$\therefore T = T_A + T_S = \underline{3.5315}$ KN-m → (5)

From (4) & (5), max torque carrying capacity is given as $\underline{\underline{2.978 \text{ KN-m}}}$

source: diginotes.in

II - IA MOM(15ME34)

1. a. Ratio of longitudinal stress to hoop stress of a thin cylinder (2M)

Sol: For a thin cylinder,

Longitudinal stress = $\sigma_L = \frac{PD}{4t}$

Hoop stress = $\sigma_c = \frac{PD}{2t}$

∴ Ratio of longitudinal stress to hoop stress is given = $\frac{\sigma_L}{\sigma_c} = \left[\frac{PD}{4t}\right] / \left[\frac{PD}{2t}\right]$

$$\frac{\sigma_L}{\sigma_c} = \frac{1}{2}$$

(or) $$\frac{\sigma_c}{\sigma_L} = 2$$

b. Prove that the volumetric strain in thin cylinder is $\frac{\delta V}{V} = (5 - 4\mu)\frac{PD}{4tE}$, with usual notation.

Sol: For a thin cylinder, change in volume $\frac{\delta V}{V}$.

$$\varepsilon_V = \frac{\delta V}{V} = (5 - 4\mu)\frac{PD}{4tE}$$

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source: diginotes.in

When the cylindrical pressure vessel is subjected to inter pressure p, there will be change in Dia, length & hence volume will be increased.

Based on Hook's law,

⇒ Circumferential strain

$$\varepsilon_c = \frac{\sigma_c}{E} - \frac{\sigma_L}{E}\mu$$

$$\varepsilon_c = \frac{PD}{2tE}\left(1 - \mu \times \frac{1}{2}\right) \longrightarrow (1)$$

$$\varepsilon_c = \frac{\delta d}{d} \longrightarrow (2)$$

$$\therefore \varepsilon_c = \frac{\delta d}{d} = \left(1 - \frac{\mu}{2}\right) \longrightarrow (3)$$

⇒ Longitudinal strain

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu\frac{\sigma_c}{E} = \frac{PD}{4tE} - \mu\frac{PD}{2tE}$$

$$\varepsilon_L = \frac{\delta L}{L} = \frac{PD}{2tE}\left(\frac{1}{2} - \mu\right) \longrightarrow (4)$$

⇒ Volume of cylinder: $V = \frac{\pi D^2 L}{4}$

$$\Rightarrow \delta V = \frac{\pi D^2 \delta L}{4} + \frac{\pi}{4} 2 \cdot D \cdot \delta D \cdot L$$

$$\therefore \frac{\delta V}{V} = \frac{\delta L}{L} + 2 \cdot \frac{\delta d}{d}$$

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$$\Rightarrow \frac{\delta V}{V} = \varepsilon_L + 2 \cdot \varepsilon_c \longrightarrow (5)$$

Substituting (3) & (4) in (5)

$$\Rightarrow \frac{\delta V}{V} = \frac{PD}{2tE}\left(\frac{1}{2} - \mu\right) + 2\left(\frac{PD}{2tE}\right)\left(1 - \frac{\mu}{2}\right)$$

$$\boxed{\frac{\delta V}{V} = \frac{PD}{4tE}\left(5 - 4\mu\right)}$$ Hence Proved

2) a) State Assumptions in the analysis of thick Cylinder

A: Assumptions

- Cylinder subjected to both pressures Internal as well as external pressures due to the considerable thickness
- Body or cylinder is homogeneous.
- Cylinder is isotropic in nature.
- Closed cylinders subjected to σ_c, P, σ_L
- Cylinder with open ends is not subjected to σ_L
- σ_L is uniform throughout the cylinder thickness
- P & σ_c vary parabolically across the section of the thickness of they cylindrical wall.

source: diginotes.in

⇒ Internal surface subjected to max pr.

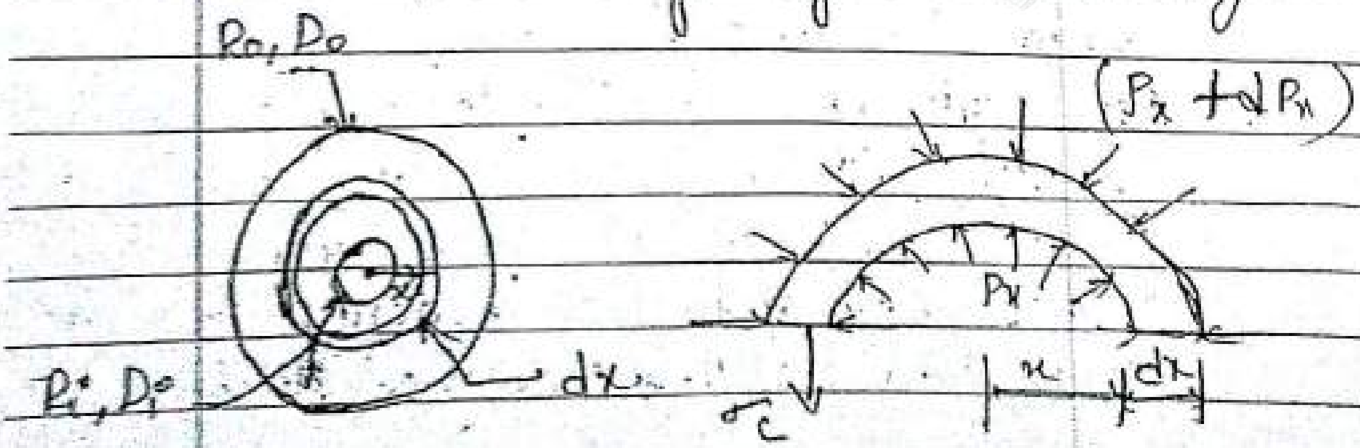
→ Magnitude of σ_c induced @ any point in the cylinder wall is always greater than that of radial stress @ same point.

→ Cylinder subjected to internal pressure,
σ_c = tensile in nature.
P = Radial stress or P_r will be compressive in nature

→ Cylinder subjected to External Pr,
σ_c & P will be under compression

b) Derive Lame's Equation & obtain expressions for Lame's Constants [06M].

For a thick Cylinder, let
L = length of cylinder
t = thickness.
D_i, D_o = Internal & Outer diameters.
P_x (P_x+dP_x) : Pressure Intensities @ Inner & Outer surface of a elemental ring



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Bursting force $= F_B = P_x \times 2xL - (P_x + dP_x) \times 2L \times (x + dx)$

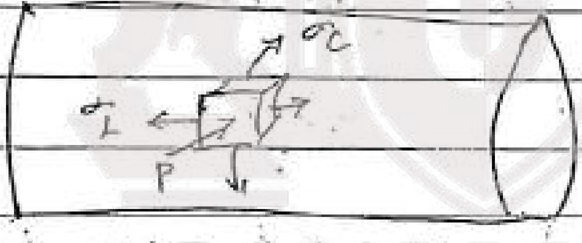
Resisting force $= F_R = \sigma_c \times 2dx\,L$

Equilibrium equation $F_B = F_R$ & ignoring higher order terms like $2 \cdot dP_x \cdot dx \cdot L$, we get

$$\sigma_c\, 2 \cdot dx \cdot L = -2P_x\, dx\, L - 2 \cdot dP_x \cdot x \cdot L$$

Dividing throughout by $2L$.

$$\sigma_c + x \cdot \frac{dP_x}{dx} + P_x = 0 \rightarrow (1)$$



← Cylinder with stresses.

Applying Generalised Hooke law for cylinder, subjected to σ_L, σ_c & Radial Pressure P_x.

Longitudinal strain $= \varepsilon_1 = \frac{\sigma_L}{E} - \frac{\mu}{E}(\sigma_c - P_x) \rightarrow (2)$

According to Lami's theory, ε_L = constant &

σ_L = constant.

$\therefore \sigma_c - P_x$ = constant

$\therefore$ from (2), $\sigma_c - P_x$ = constant.

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let $\sigma_c - P_x = 2a$

$\sigma_c = 2a + P_x \longrightarrow (3)$

Sub (3) in (1) $\Rightarrow$ $P_x + 2a + x \cdot \frac{dP_x}{dx} + P_x = 0$

$\therefore \; -2(P_x + a) = x \cdot \frac{dP_x}{dx}$

$\Rightarrow \frac{dP_x}{P_x + a} = -2 \cdot \frac{dx}{x}$

Integrating above we get

$\log_e (P_x + a) = -2 \log_e x + \text{constant}$

$\log_e (P_x + a) = -2 \log_e x + \log_e b$

$\log (P_x + a) = \log_e x^{-2} + \log_e b$

$P_x + a = b/x^2$

$\therefore \log(ab) = \log a + \log b$

$\log x^{-2} = \log\left(\frac{1}{x^2}\right)$

$$\boxed{\therefore P_x = -a + b/x^2}$$

Sub in (1), $\boxed{\sigma_c = +a + b/x^2}$

$\therefore$ Lami's constants a, b dependent on BC & above 2 equations are called Lami's Equation

source: diginotes.in

3) A thin cylindrical shell of 0.6 m dia & 0.9 m Long subjected to $P_i = 1.2\ N/mm^2$ & $t = 15$ mm. Determine
(a) σ_L, σ_c, τ 4M
(b) δd, δL, δV 4M.

Sol: Given data

$D = 0.6$ m $P = 1.2\ N/mm^2$
$L = 0.9$ m $E = 200$ GPa
$t = 15$ mm $\mu = 0.3$

(a) $\sigma_L = \frac{PD}{4t} = \frac{1.2 \times 600}{4 \times 15} = 12$ MPa

$\sigma_c = \frac{PD}{2t} = \frac{1.2 \times 600}{2 \times 15} = 24$ MPa

$\tau = \frac{\sigma_c - \sigma_L}{2} = \frac{24 - 12}{2} = 6$ MPa

(b) $\delta d = (e_d) \times d$

$e_d = \varepsilon_c = \frac{\sigma_c}{E} - \mu \cdot \frac{\sigma_L}{E}$

$= \frac{24}{2 \times 10^5} - \frac{0.3 \times 12}{2 \times 10^5} = 1.02 \times 10^{-4}$

$\therefore \delta d = \varepsilon_c \cdot d = 1.02 \times 10^{-4} \times 600 = \underline{0.0612\ mm}$

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$\Rightarrow$ change in length $= \delta l = \varepsilon_l \cdot L$

But $\varepsilon_l = \frac{\sigma_l}{E} - \mu \cdot \frac{\sigma_c}{E} = 2.4 \times 10^{-5}$

$\Rightarrow \delta l = 2.4 \times 10^{-5} \times 900 = \underline{0.0216 \text{ mm}}$

$\Rightarrow \delta V = V \cdot \varepsilon_v$

$\varepsilon_v = \varepsilon_l + 2\varepsilon_c = 2.28 \times 10^{-4}$

$\delta V = 2.28 \times 10^{-4} \times \left[\frac{\pi \times 600^2}{4} \times 900\right]$

$\therefore \delta V = \underline{58 \times 10^3 \text{ mm}^3}$

(4) A thick cylinder with $D_i = 80$ mm, $D_o = 120$ mm subjected to $P_o = 40$ kN/m², $P_i = 10$ KPa

(a) Calculate σ_c, Radial stress @ external & internal

(b) Plot variations.

Sol. Given data $D_i = 80$ mm $\quad R_i = 40$ mm

$D_o = 120$ mm $\quad R_o = 60$ mm

$P_o = 40$ kN/m² $\quad P_i = 10$ kN/m²

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$$P = \frac{b}{x^2} - a$$

$$P_0 = \frac{b}{x_0^2} - a = \frac{b}{(60 \times 10^{-3})^2} - a = 40 \times 10^3$$

$$P_i = \frac{b}{x_i^2} - a = \frac{b}{(40 \times 10^{-3})^2} - a = 10 \times 10^3$$

$$b\left(\frac{1}{(60 \times 10^{-3})^2} - \frac{1}{(40 \times 10^{-3})^2}\right) = 30 \times 10^3$$

$b =$

$\therefore a =$

$$\sigma_{ci} = \frac{b}{x_i^2} + a, \quad \sigma_{co} = \frac{b}{x_0^2} + a$$

$\sigma_{ci} =$ N/m² $\quad$ $\sigma_{co} =$ N/m²

$P_i =$ N/m² $\quad$ $P_0 =$ N/m²

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(5) (a) A thick cylindrical of $d_i = 200$ mm subjected to an $P_i = 7\ N/mm^2$, It tensile stress in shell is $8\ N/mm^2$,

(a) Find Lame's constant

(b) Also find thickness.

Sol: Given data.

$d_i = 200$ mm

$P_i = 7\ N/mm^2$

$\sigma_a = 8\ N/mm^2$

The cylinder is subjected to maximum hoop stress @ the inner wall.

$\therefore\ \sigma_a = \sigma_c$ @ inner wall

$\Rightarrow \sigma_a = \sigma_{ci}$

BC's $\Rightarrow$ (i) $P_i = 7\ N/mm^2$ @ $R_i = 100$ mm

(ii) $\sigma_{ci} = 8\ N/mm^2$ @ $R_i = 100$ mm

① $P = \frac{b}{x^2} - a \Rightarrow 7 = \frac{b}{100^2} - a$

② $\sigma = \frac{b}{x^2} + a \Rightarrow 8 = \frac{b}{100^2} + a$

$15 = \frac{2b}{100^2}$

$\Rightarrow b =$

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$\therefore$ a =

(b) Sub $P_o = 0$ @ $r = R_o$ in (1) since cylinder is not subjected to any external Pr.

$0 = - + \frac{}{R_o^2}$

$\therefore R_o = $ mm.

$\therefore t = R_o - R_i = $ mm.

(b)
(a) Explain Sagging & Hogging Bending Moments 2
(b) Relationship b/w Load intensity, Shear Force & BM 4
(c) Explain various types of loads with sketch. 2

Sol: Sagging Bending Moment:

The moment causing a portion of the beam to be bent into concave shape on its top surface is known as Sagging moment or Positive moment. During this the top surface will be under compression & bottom surface of the beam will be under Tension.

+ve Concave Bending Moment.

source: diginotes.in

Hogging Bending Moment:

The moment tending to bend a beam with convex shape on its top is known as Hogging moment or negative moment. During this, top or inner surface of beam will be under compression & top and outer surface will be under Tension.

(b) Relation b/w Load Intensity, SF & BM

Let, w(x) = load distributed on beam

dx = Elemental length

R_1, R_2 = Support @ left & right side.

F = Shear force } on left side face.

M = Bending Moment }

F+dF, M+dM = Shear force & BM on Right side face.

Under equilibrium condition

$\Sigma F_y = 0$

$\therefore F - (F + dF) - w\,dx = 0$

$\frac{dF}{dx} = -w \longrightarrow (1)$

(or) $dF = w.dx$

From above equation (1), it can be seen that the "slope of shear force diagram is equal to load intensity".

-ve sign indicates the shear force decreases with increase in x, when distributed load acts downwards.

Considering elemental portion b/w x_1 & x_2,

$$\int_{x_1}^{x_2} w.dx = \int_{F_1}^{F_2} dF = F_2 - F_1 \longrightarrow (2)$$

(2) indicates, "change in SF between any 2 sections is equal to area under the load diagram b/w the 2 sections".

Taking moment about left side edge, we get

$\Sigma M_o = 0$

$$M - (M + dM) + (F + dF)dx + w\,dx \cdot \frac{dx}{2} = 0$$

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Ignore higher order terms, we get

$$F = \frac{dM}{dx} \longrightarrow (5) \quad dM = F \cdot dx$$

from (5), "The SF @ any section is equal to the slope of B.M.D (rate of change of BM) @ that section."

On Integrating above equation,

$$\int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} F \, dx = M_2 - M_1$$

∴ The change in moment b/w any 2 sections is equal to the area under shear force diagram b/w the 2-sections.

(7). Types of Loads

1) Simply applied load or Point load.
2) Distributed load
   - ↳ Uniformly Distributed load
   - ↳ Uniformly Varying load.
3) Inclined load

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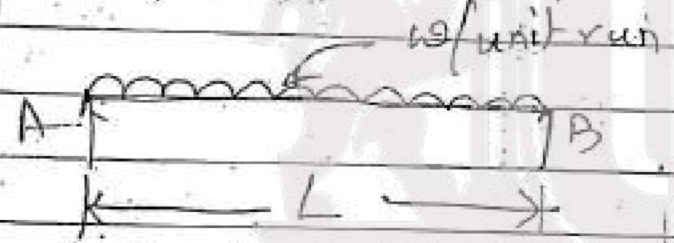
1) Point load: If any load acting on a beam is concentrated @ a point as shown below then it is called Point or concentrated load.

Fig: Simply supported beam with point load.

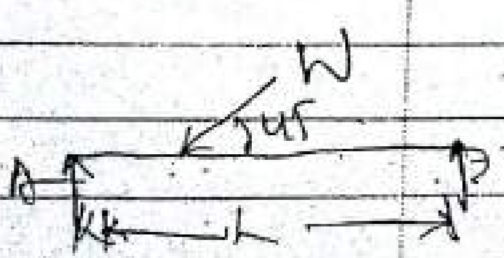
2) Distributed load: The amount of load that is distributed either uniformly or varyingly throughout the length of the beam, then it is called Distributed load.

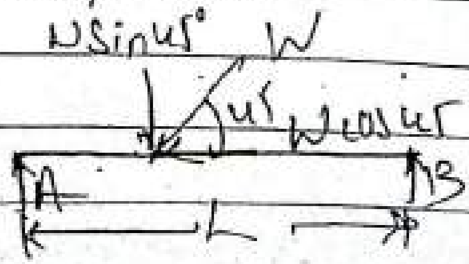


Uniformly Distributed load.

Varying load.

3). Inclined load: load which is inclined on the beam as shown below is known as Inclined load. It should be divided into 2 components & named as Horizontal & Vertical components.





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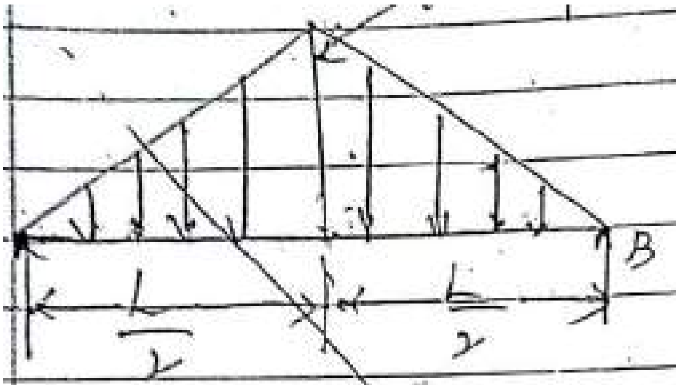
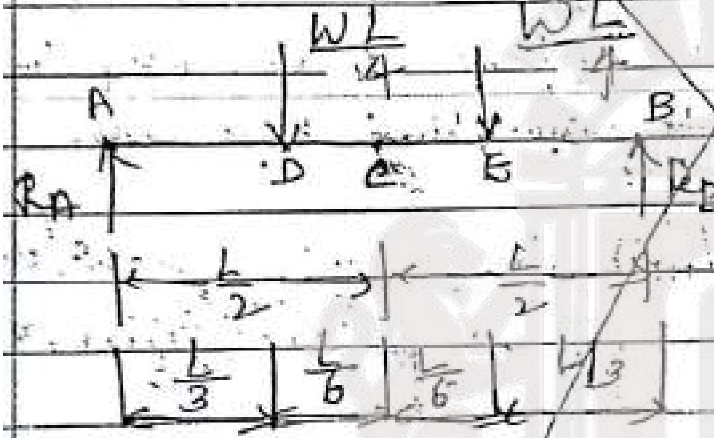


(a) free body dia 2
(b) SFD 2
(c) BMD 4

free body diagram

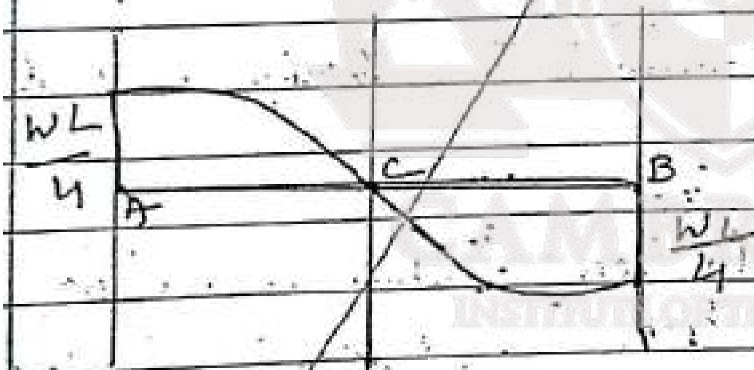


Point load $= w \times \frac{L}{2} \times \frac{1}{2}$

acting @ $= L/3$ from A & B

$R_A + R_B = WL/2$

$R_A = R_B = \frac{WL}{4}$



SF @ B $= \frac{-WL}{4}$

E = 0

C =

D = WL/2

A =

BM @ B = 0 & A = 0

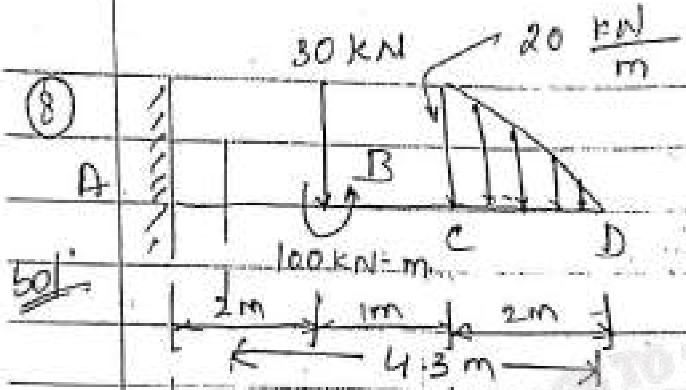
E = $\underline{WL^2}$

C =

D =

source: diginotes.in

(8)



(a) Draw SFD

(b) Draw BMD

Sol:

Support Reactions

$R_A = 50$ KN

$M_A = -33.33$ KN-m

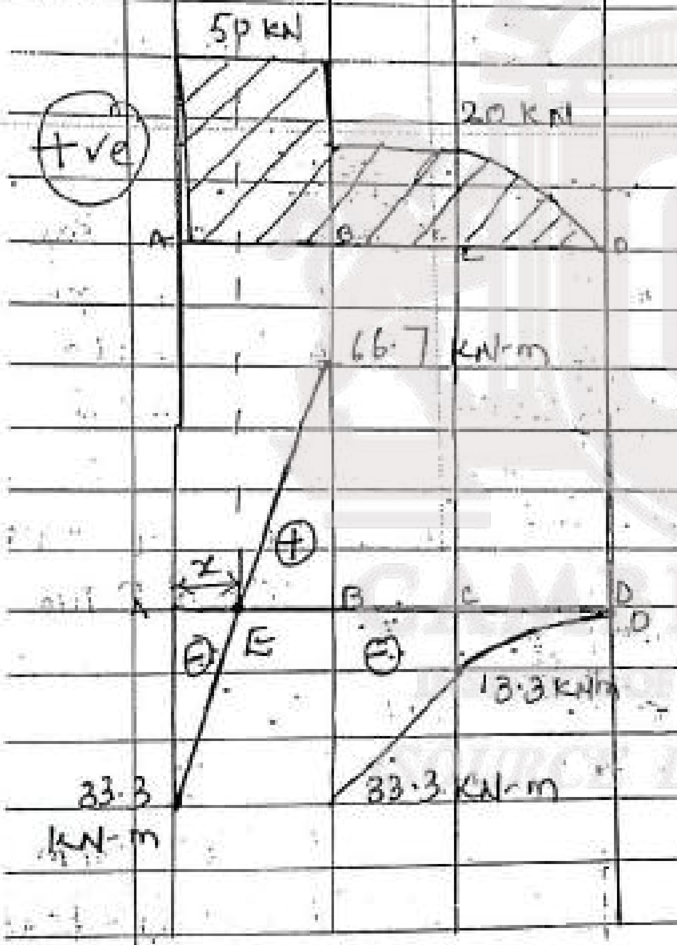
SF Diagram

SF @ B = 0

C = $20 \times 2 \times \frac{1}{2} = 20$ KN

B = 20 + 30 = 50 KN

A = 20 + 30 = 50 KN



BM - Diagram

BM @ D = 0 KN-m

@ C = $-20 \times \frac{2}{3} = -13.3$

= −13.3 KN-m

@ B = $-20 \times (\frac{2}{3} + 1)$

= −33.3 KN-m

@ B = −33.3 + 100

= 66.7 KN-m

@ A = $-20 \times (\frac{2}{3} + 3) - 30 \times 2 + 100$

= −33.3 KN-m

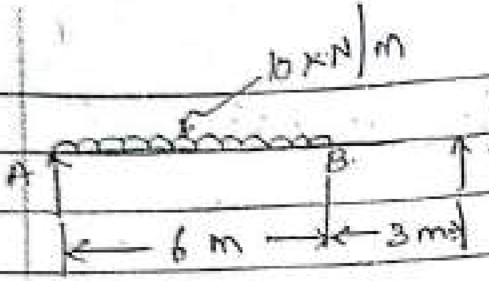
Point E = Point of Contraflexure.

it is @ a distance of x from A.

x = 0.7 (or) 4.3 m from B.

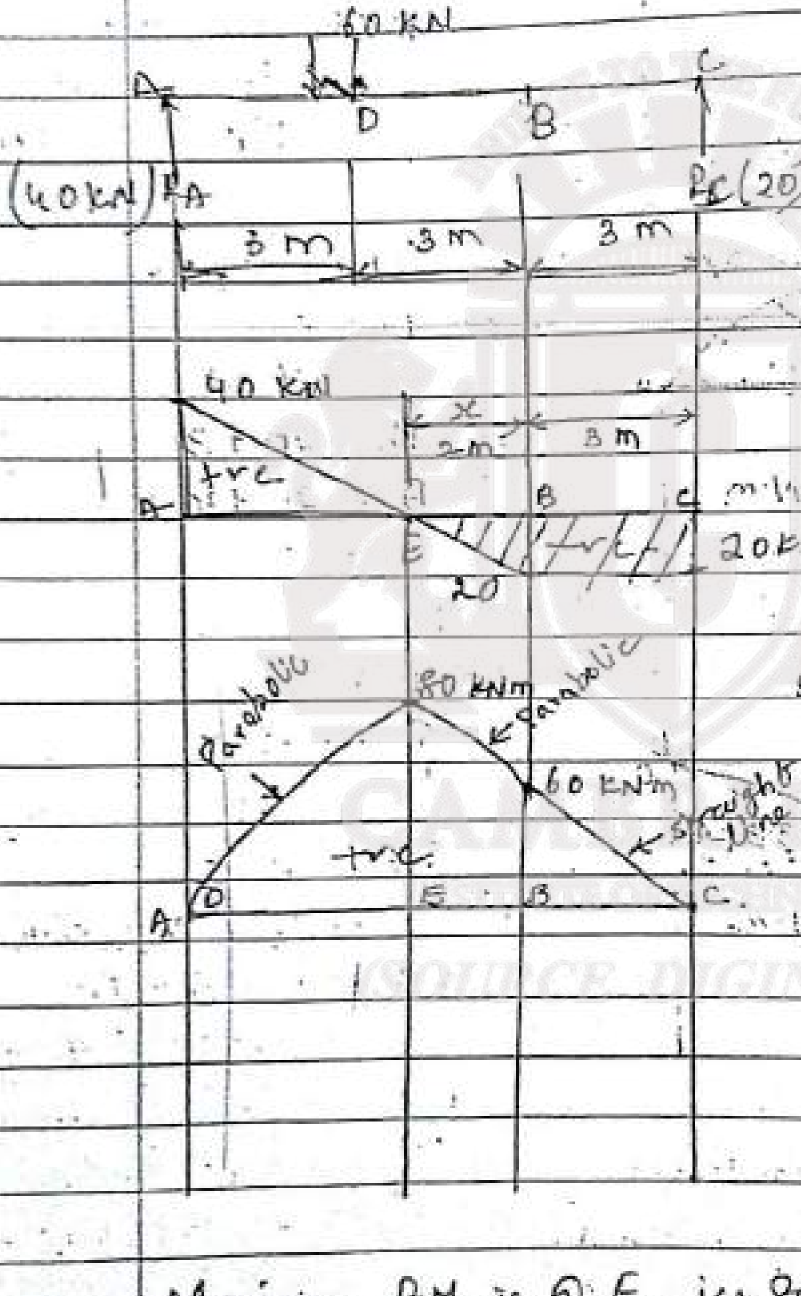
source: diginotes.in

(9)



Support reactions

Sol. Free body diagram



$R_A + R_C = 60$ kN

$M_A = 0$

$0 = R_C \times 9 - 60 \times 3$

$R_C = 20$ kN

$\therefore R_A = 40$ kN

SFD

SF @ C = +20 kN

B = −20 kN

A = −20 + 60 = 40 kN

A = −40 + 40 = 0 kN

SF @ E = 0 = −20 + 10 × x

x = 2 m from B

BMD

BM @ C = 0

B = 60 kNm

$E = 20 \times 5 - 10 \times 2 \times \frac{2}{2}$

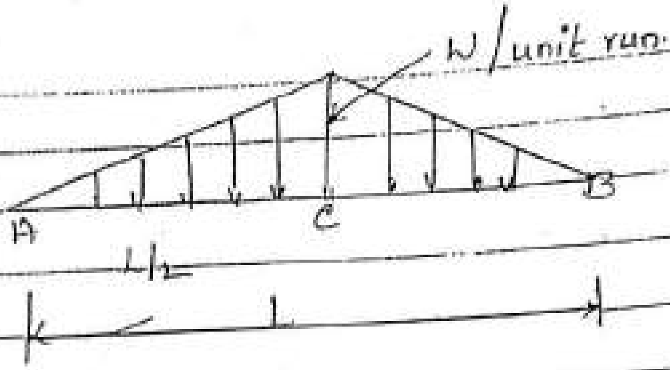
= 80 kNm

A = 0

Maximum BM is @ E i.e. 80 kNm.

source: diginotes.in

Q7



Sol: Total load on beam = Area of load diagram ABD

$$= \frac{AB \times CD}{2} = \frac{L \times w}{2} \text{ or } \frac{wL}{2}$$

$R_A + R_B$ = Load = wL

$$R_A = R_B = \frac{1}{2}\left(\frac{wL}{2}\right)^2 = \frac{wL}{4}$$

Consider any section X-X b/w A & C @ a distance 'x' from end A. The rate of loading @ X

= Vertical distance XD in load diagram

$$\left.\begin{matrix} L/2 = w \\ x = ? \end{matrix}\right\} \Rightarrow \frac{wx}{(L/2)} = \frac{2wx}{L}$$

load on length AX = Area of load diagram AXD

$$= \frac{x \cdot XD}{2} = \frac{x \cdot \left(\frac{2w}{L}\right)x}{2}$$

$$= \frac{wx^2}{L} \text{ acting @ } \frac{x}{3} \text{ from X}$$

∴ SF @ X-X is given by

F_x = R_A − load on length AX

$$= \frac{wL}{4} - \frac{wx^2}{L} \rightarrow (1)$$

source: diginotes.in

Equa^n (i) shows, SF varies parabolically b/w A & C.

@ A, $x=0$, hence $F_x = \frac{WL}{4} - \frac{W}{L} \cdot 0 = \frac{WL}{4}$

@ C, $x = L/2$ hence $F_C = \frac{WL}{4} - \frac{W}{L}\left(\frac{L}{2}\right)^2 = 0$

@ B, $F_B = R_B = -\frac{WL}{4} = 0$

BM diagram :-

BM @ A = 0

@ B = 0

@ X = $M_x = (R_A \cdot x) - (\text{load of length } AX)\left(\frac{x}{3}\right)$

$= \frac{WL}{4} x - \frac{Wx^2}{L} \cdot \frac{x}{3}$

$\frac{WLx}{4} - \frac{Wx^3}{3L}$ — (ii)

(ii) shows that BM b/w A & C varies accoring to cubic law.

@ A, $x=0$, $M_A = 0$

@ C, $x = L/2$ $M_C = \frac{WL}{4} \cdot \frac{L}{2} - \frac{W}{3L}\left(\frac{L}{2}\right)^3$

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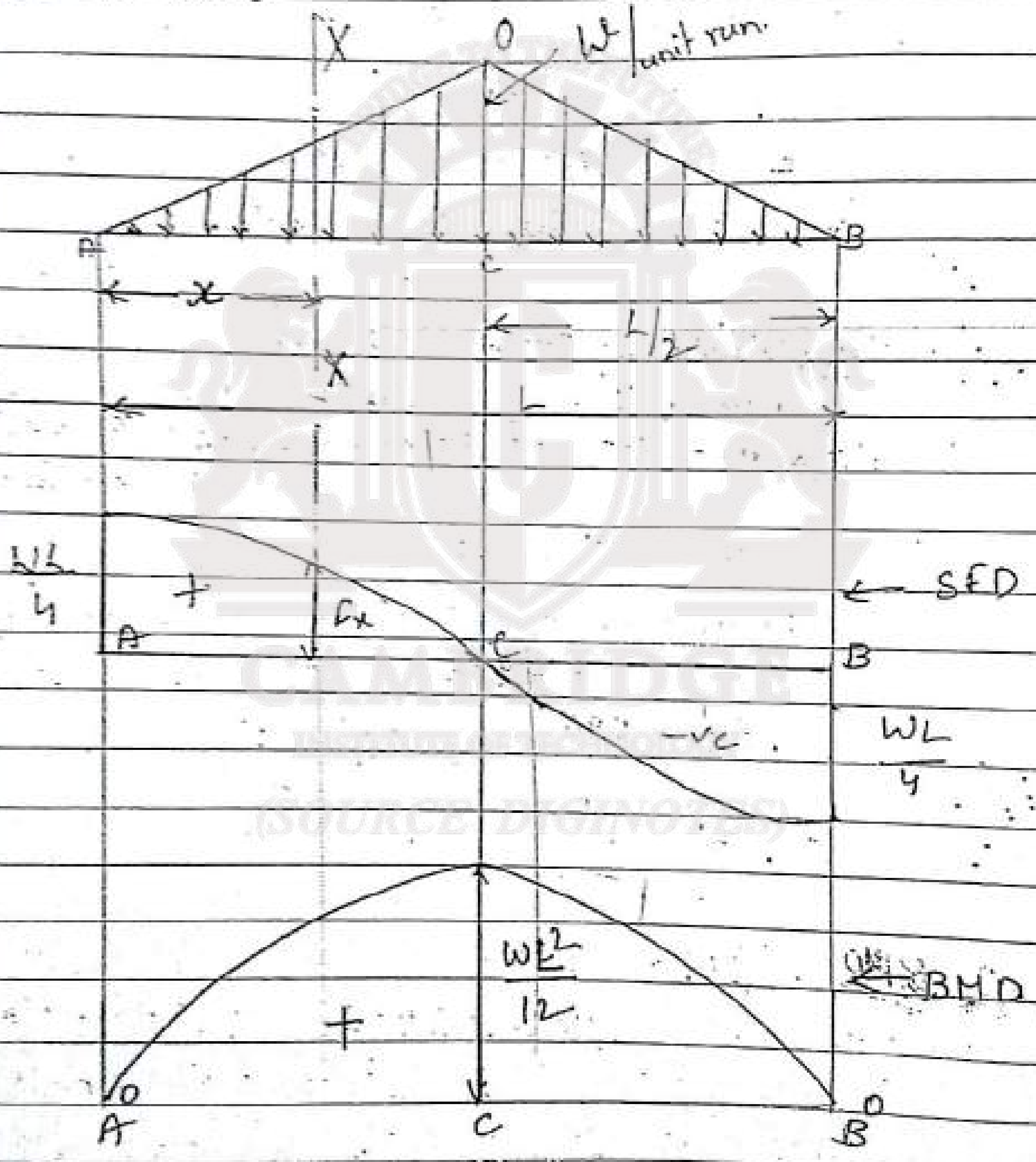
$$= \frac{WL^2}{12}$$

∴ Max BM occurs @ centre of the beam, where SF is zero



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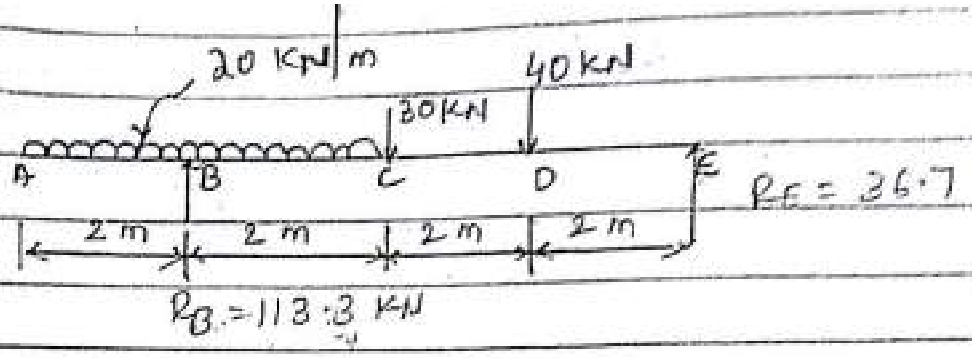
(i)



Support Reactions: $\Sigma F_y = 0$

$R_E + R_B = 40 + 30 + 20\times2 + 20\times2$

$R_E + R_B = 150$ KN → (1)

Taking Moment @ B on both sides

∴ M_B @ L.H.S = M_B @ RHS

$R_E \times 6 - 40\times4 - 30\times2 - 20\times2\times\left(\frac{2}{2}\right)$ = LHS

$- 20\times2\times\frac{2}{2}$ = RHS

⇒ $6R_E - 260 = -40$

$6R_E = -40 + 260 = 220$

$R_E = \frac{220}{6} = 36.7$ KN

∴ $R_B = 113.3$ KN

BMD: BM @ E = 0

D = $36.7\times2 = 73.4$ KNm

C = $36.7\times4 - 40\times2 = 66.8$ KNm

R.H.S. of beam, B = $36.7\times6 - 40\times4 - 30\times2 - 20\times2\times1 = -39.8$ KNm

A = $36.7\times8 - 40\times6 - 30\times4 - 20\times2(3)$

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$+ R_B \times 2 - 20 \times 2 \times 1 = 0$

BM @ B from L.H.S = $-20 \times 2 \times 1 = -40$ kNm.

SFD : SF @ E = −36.7 kN

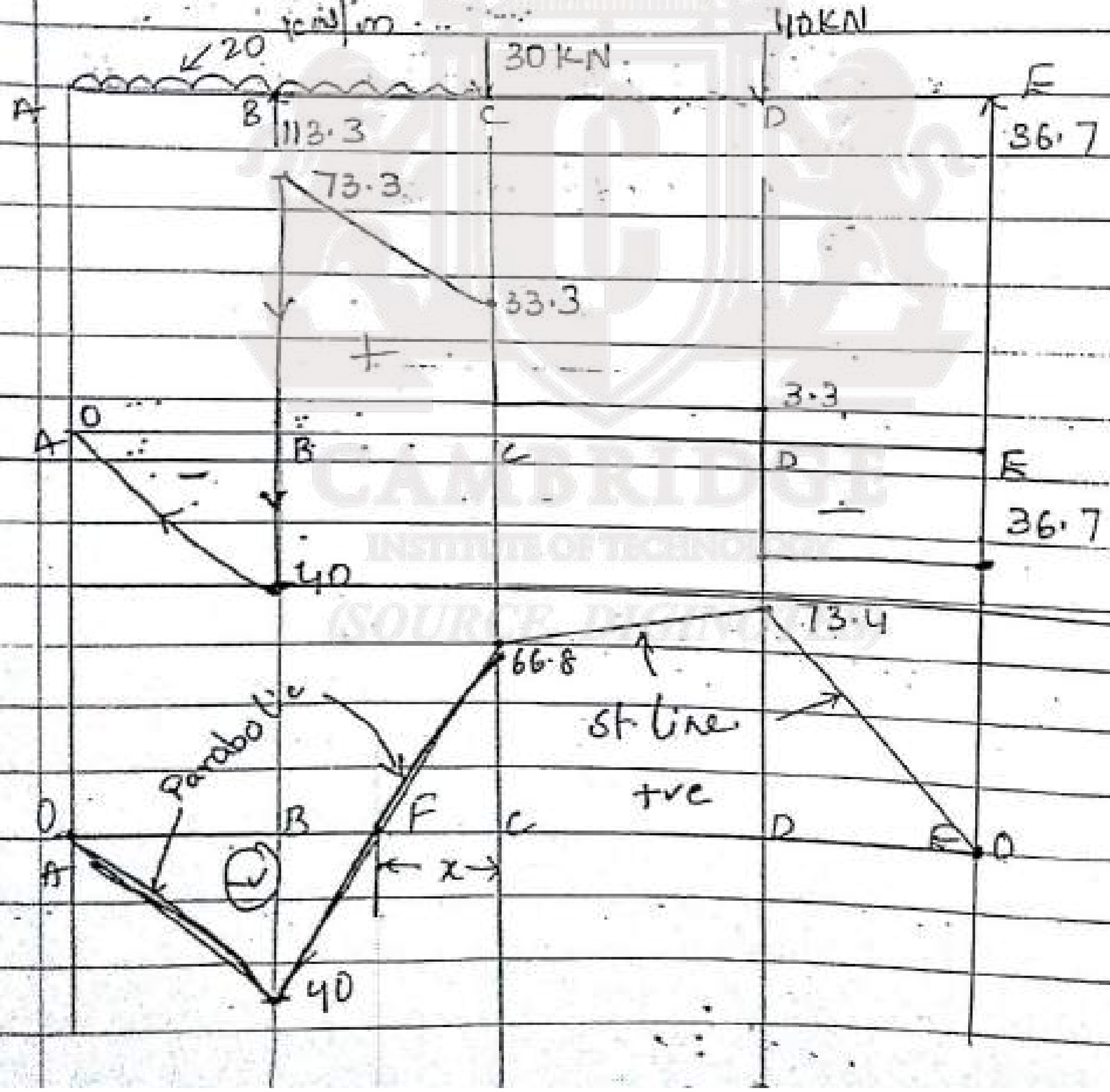
D = 40 − 36.7 = 3.3 kN

C = 33.3 kN.

B = 33.3 + (20×2) − 113.3

= −40 kN

A = −40 + 40 = 0



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Let F be point of contraflexure and it is @ a distance of x+2+2 from E.

To calculate x = ?

Taking Moment @ x = 0

$M_x = 0 = 36.7 x (4 + x) - 40 x (2 + x)$

$- 30 x x - 20 x x x \frac{x}{2}$

$0 = 146.8 + 36.7x - 80 - 40x - 30x$

$-10x^2$

$146.8 - 33.3x - 10x^2 - 80$

$x = 0.98\ m$

source: diginotes.in

Module 3

Bending Moment and Shear Force

Objectives:

Determine the shear force, bending moment and draw shear force and bending moment diagrams, describe behaviour of beams under lateral loads. Stresses induced in beams, bending equation derivation & Deflection behaviour of beams

Learning Structure

- 3.1 Types Of Beams
- 3.2 Shear Force
- 3.3 Bending Moment
- 3.4 Shear Force Diagram And Bending Moment
- 3.5 Relations Between Load, Shear And Moment
- 3.6 Problems
- 3.7 Pure Bending
- 3.8 Effect Of Bending In Beams
- 3.9 Assumptions Made In Simple Bending Theory
- 3.10 Problems
- 3.11 Deflection Of Beams
- Outcomes
- Further Reading

3.1 TYPES OF BEAMS

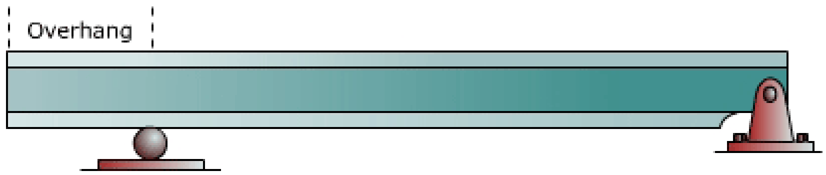
a) Simple Beam

A simple beam is supported by a hinged support at one end and a roller support at the other end.

b) Cantilever beam

A cantilever beam is supported at one end only by a fixed support.

c) Overhanging beam.



An overhanging beam is supported by a hinge and a roller support with either or both ends extending beyond the supports.

Note: All the beams shown above are the statically determinate beams.

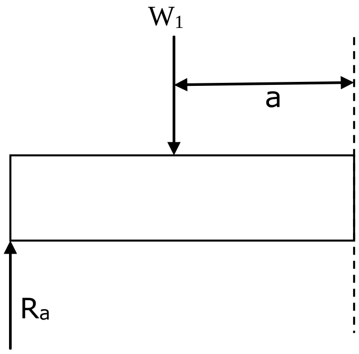


Fig 2 :Shear Force

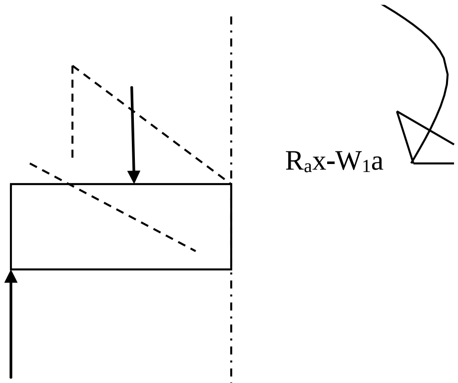


Fig 3 : Bending Moment

Consider a simply supported beam subjected to loads W_1 and W_2. Let R_A and R_B be the reactions at supports. To determine the internal forces at C pass a section at C. The effects of R_A and W_1 to the left of section are shown in Fig (b) and (c). In each case the effect of applied load has been transferred to the section by adding a pair of equal and opposite forces at that section. Thus at the section, moment M = (W_1a-R_ax) and shear force F = (R_A-W_1), exists. The moment M which tend to bends the beam is called bending moment and F which tends to shear the beam is called shear force.

Thus the resultant effect of the forces at one side of the section reduces to a single force and a couple which are respectively the vertical shear and the bending moment at that section. Similarly, if the equilibrium of the right hand side portion is considered, the loading is reduced to a vertical force and a couple acting in the opposite direction. Applying these forces to a free body diagram of a beam segment, the segments to the left and right of section are held in equilibrium by the shear and moment at section.

Thus the shear force at any section can be obtained by considering the algebraic sum of all the vertical forces acting on any one side of the section

Bending moment at any section can be obtained by considering the algebraic sum of all the moments of vertical forces acting on any one side of the section.

3.2 Shear Force

It is a single vertical force developed internally at any point on the beam to balance the external vertical forces and keep the point in equilibrium. It is therefore equal to algebraic sum of all external forces acting to either left or right of the section.

3.3 Bending Moment

It is a moment developed internally at each point in a beam that balances the external moments due to forces and keeps the point in equilibrium. It is the algebraic sum of moments to section of all forces either on left or on right of the section.

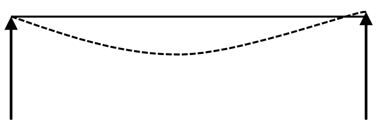
3.3.1 Types of Bending Moment

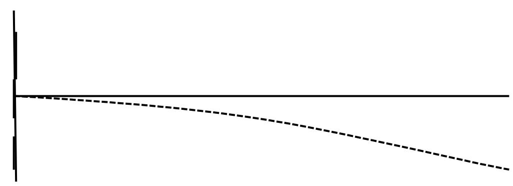
1) Sagging bending moment

The top fibers are in compression and bottom fibers are in tension.

2) Hogging bending moment

The top fibers are in tension and bottom fibers are in compression.





Sagging Bending Moment Hogging Bending Moment

3.4 Shear Force Diagram and Bending Moment

3.4.1 Diagram Shear Forces Diagram (SFD)

The SFD is one which shows the variation of shear force from section to section along the length of the beam. Thus the ordinate of the diagram at any section gives the Shear Force at that section.

3.4.2 Bending Moment Diagram (BMD)

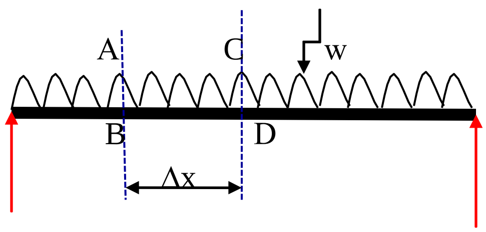
The BMD is one which shows the variation of Bending Moment from section to section along the length of the beam. The ordinate of the diagram at any section gives the Bending Moment at that section.

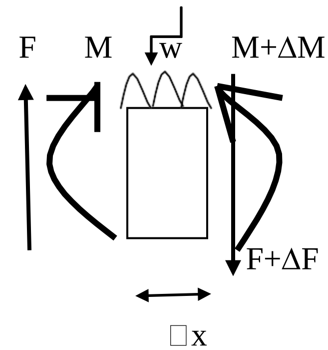
3.4.3 Point of Contraflexure

When there is an overhang portion, the beam is subjected to a combination of Sagging and Hogging moment. The point on the BMD where the nature of bending moment changes from hogging to sagging or sagging to hogging is known as point of contraflexure. Hence, at point of contraflexure BM is zero. The point corresponding to point of contraflexure on the beam is called as point of inflection.

3.5 RELATIONS BETWEEN LOAD, SHEAR AND MOMENT

Consider a simply supported beam subjected to a Uniformly Distributed Load w/m. Let us assume that a portion PQRS of length Δx is cut and taken out. Consider the equilibrium of this portion





$\sum V=0$

$F - (F+\Delta F) - w\,\Delta x = 0$

Limit $\Delta x \to 0$, then or F =

Taking moments about section CD for equilibrium

$$M-(M+\Delta M)+F\,\Delta x-(w(\Delta x)^2/2)=0$$

Rate of change of Shear Force or slope of SFD at any point on the beam is equal to the intensity of load at that point.

Properties of BMD and SFD

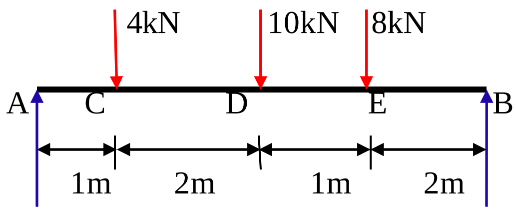
1) when the load intensity in the region is zero, Shear Force remains constant and Bending Moment varies linearly.

2) When there is Uniformly Distributed Load (UDL), Shear Force varies linearly and BM varies parabolically.

3) When there is Uniformly Varying Load (UVL), Shear Force varies parabolically and Bending Moment varies cubically.

3.6 Problems:

1. **A simply supported beam is carrying point loads, as shown in figure. Draw the SFD and BMD for the beam.**



11

7

+ve SFD

3

-ve SFD

11

25

22

11 +ve BMD

kN-m

To draw S.F.D. and B.M.D. we need R_A and R_B.

By taking moment of all the forces about point A, we get

$R_B \times 6 - (8 \times 4) - (10 \times 3) - (4 \times 1) = 0$

$R_B = 11$ kN

From condition of static equilibrium:

$R_A + 11 - 4 - 10 - 8 = 0$

$R_A = 11$ kN

Shear Force Calculations

SF at A $F_A = + R_A = + 11$ kN

SF left of C $F_C = + R_A = +11$ kN SF

right of C $F_C = + 11 - 4 = + 7$ kN

SF left of D $F_D = + 11 - 4 - 10 = 7$ kN

SF right of D $F_D = + 11 - 4 - 10 = - 3$ kN SF

left of E $F_E = + 11 - 4 - 10 = - 3$ kN

SF left of E $F_E = + 11 - 4 - 10 - 8 = - 11$ kN SF

left of B $F_B = + 11 - 4 - 10 - 8 = - 11$ kN

Bending moment Calculations

At x = 0, $M_A = 0$

At x = 1 m; $M_C = + R_A.1 = 11 \times 1 = 11$ kN m At

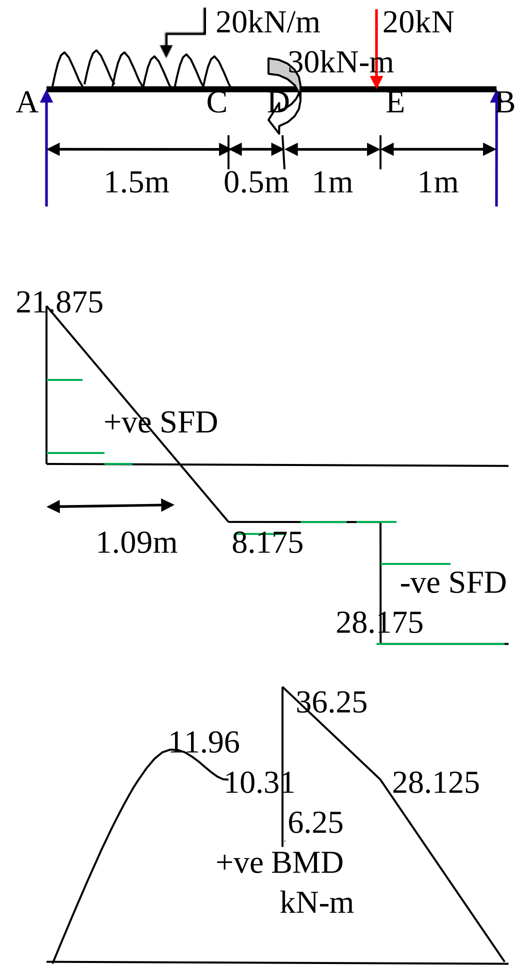
x = 3 m; $M_D = 11 \times 3 - 4 (3 - 1) = 25$ kN m

At x= 4m

$M_E = 11 \times 4 - 4 (4 - 1) - 10 (4 - 3) = 22$ kN m At

X= 6m $M_B = 0$

2. Draw the SF and BM diagram for the simply supported beam loaded as shown in fig.



To draw S.F.D. and B.M.D. we need R_A and R_B.

By taking moment of all the forces about point A, we get

$R_B \times 4 - (20 \times 3) - (20 \times 1.5^2/2) - 30 = 0$

$R_B = 28.125$ kN

From condition of static equilibrium:

$R_A + 28.125 - 30 - 20 = 0$

$R_A = 21.875$ kN

Shear Force Calculations

SF at A $F_A = + R_A = + 21.875$ kN

SF left of C $F_C = +21.875 - 30 = -8.175$ kN SF

right of C $F_C = +21.875 - 30 = -8.175$ kN SF

left of D $F_D = +21.875 - 30 = -8.175$ kN SF

right of D $F_D = +21.875 - 30 = -8.175$ kN SF

left of E $F_E = +21.875 - 30 = -8.175$ kN SF

left of E $F_E = +21.875-30-20 = -28.125$ kN SF

left of B $F_B = +21.875-30-20 = -28.125$ kN

SF is zero, between A & C at x = 1.09m

Bending moment Calculations

At x = 0, $M_A = 0$

At x = 1.5 m;

$M_C = 21.875 \times 1.5 - 20 \times 1.5^2/2 = 10.31$ kN m At x = 1.09 m;

$M_{max} = 21.875 \times 1.09 - 20 \times 1.09^2/2 = 11.96$ kN m

At x = 2 m;

$M_D = 21.875 \times 2 - 20 \times 1.5(0.75+0.5) = 6.25$ kN m

At x = 2 m;

$M_D = 21.875 \times 2 - 20 \times 1.5(0.75+0.5)+30$

$= 36.25$ kN m

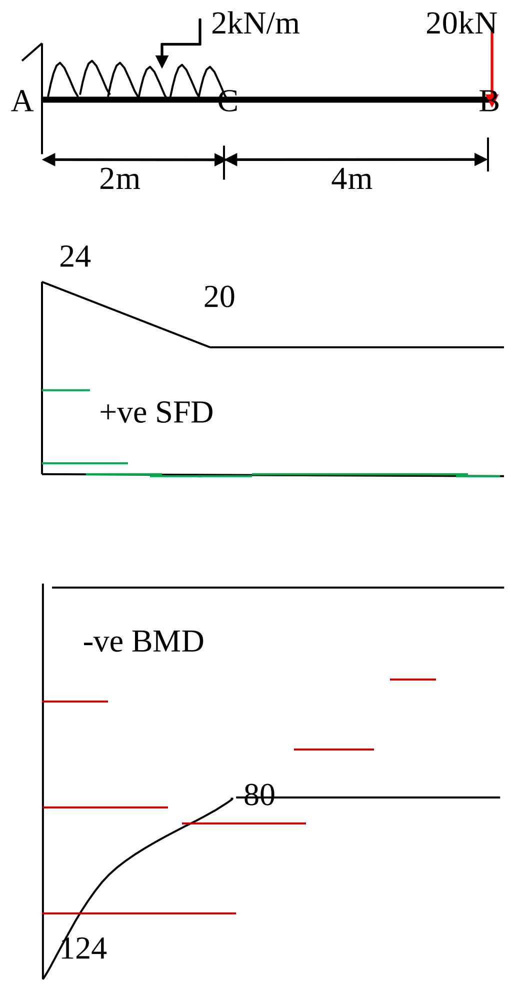
At x= 3m

$M_E = 21.875 \times 3 - 20 \times 1.5(0.75+1.5)+30$

$= 28.125$ kN m

At X= 4m $M_B = 0$

3. A cantilever is shown in fig. Draw the BMD and SFD. What is the reaction at supports?



To draw S.F.D. and B.M.D. we need R_A and R_B.

By taking moment of all the forces about point A, we get

$R_B \times 4 - (20 \times 3) - (20 \times 1.5^2/2) - 30 = 0$

$R_B = 28.125$ kN

From condition of static equilibrium:

$R_A + 28.125 - 30 - 20 = 0$

$R_A = 21.875$ kN

Shear Force Calculations

SF at B	$F_B = +20$ kN
SF right of C	$F_C = +20$ kN
SF left of C	$F_C = +20$ kN
SF right of A	$F_A = +20 + 2x\,2 = 24$ kN

Since the moment is considered from right to left the anticlockwise moments are considered to be +ve and clockwise moments are considered to be -ve

Bending moment Calculations

At x = 0, $M_B = 0$

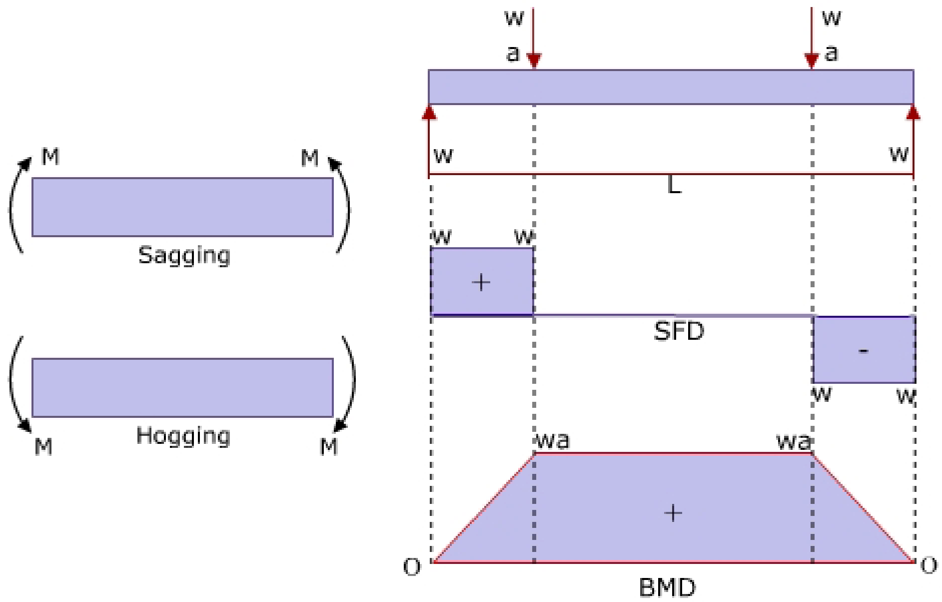
At x = 4 m; $M_c = -20x\,4 = -80$ kN-m At

x = 6 m;

$M_A = -(20 \times 6 + 2 \times 2^2/2) = 124$ kN m

Stresses in Beams

3.7 Pure Bending



A beam or a part of a beam is said to be under pure bending if it is subjected to only Bending Moment and no Shear Force.

3.8 Effect of Bending in Beams

The figure shows a beam subjected to sagging Bending Movement. The topmost layer is under maximum compressive stress and bottom most layer is under maximum tensile stress. In between there should be a layer, which is neither subjected to tension nor to compression. Such a layer is called "Neutral Layer". The projection of Neutral Layer over the cross section of the beam is called "Neutral Axis".

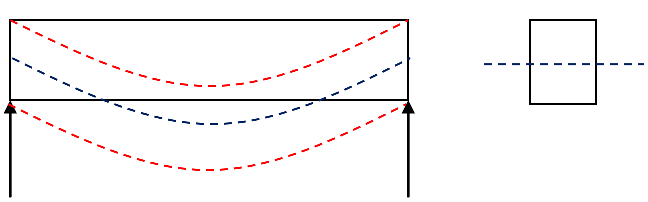


Fig-1

When the beam is subjected to sagging, all layers below the neutral layer will be under tension and all layers above neutral layer will be under compression. When the beam is subjected to hogging, all layers above the neutral layer will be under tension and all the layers below neutral layer will be under compression and vice versa if it is hogging bending moment

3.9 Assumptions made in simple bending theory

- The material is isotropic and homogenous.
- The material is perfectly elastic and obeys Hooke's Law i.e., the stresses are within the limit of proportionality.
- Initially the beam is straight and stress free.
- Beam is made up of number of layers and they undergo bending independently.
- Bending takes place over an arc of a circle and the radius of curvature is very large when compared to the dimensions of the beam.
- Normal plane sections before bending remain normal and plane even after bending.
- Young's Modulus of Elasticity is same under tension a ndcompression.

3.9.1 Euler- Bernoulli bending Equation (Flexure Formula)

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

where,

M = Resisting moment developed inside the material against applied bending movement and is numerically equal to bending moment applied (Nmm)

I = Moment of Inertia of cross section of beam about the Neutral Angle. (mm^4)

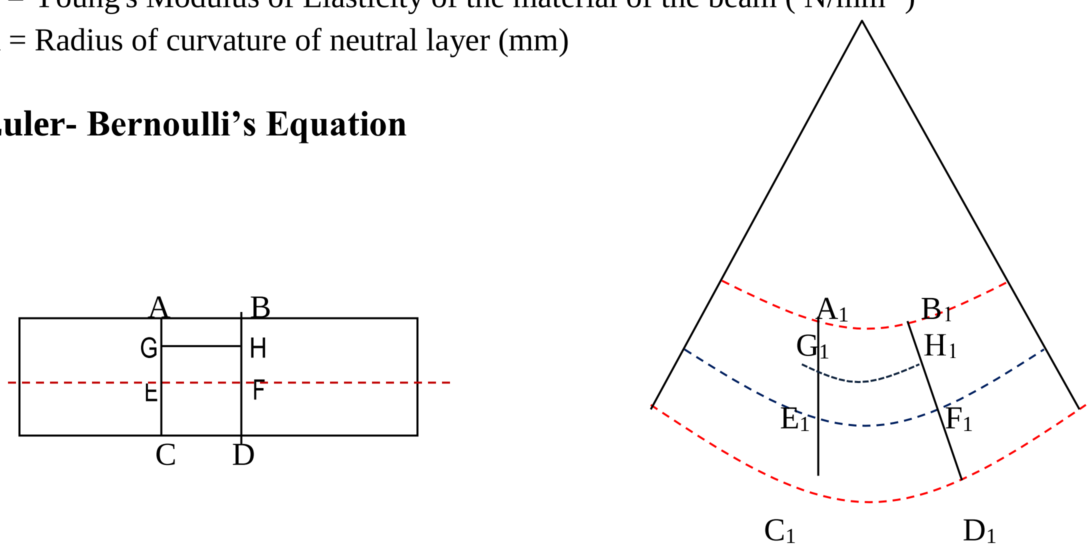
F = Direct Stress (Tensile or Compression) developed in any layer of the beam (N/mm^2)

Y = Distance of the layer from the neutral axis (mm)

E = Young's Modulus of Elasticity of the material of the beam (N/mm^2)

R = Radius of curvature of neutral layer (mm)

Euler- Bernoulli's Equation



Consider two section very close together (AB and CD). After bending the sections will be at A_1 B_1 and C_1 D_1 and are no longer parallel. AC will have extended to A_1 C1 and B_1 D_1 will have compressed to B_1D_1. The line EF will be located such that it will not change in length. This surface is called neutral surface and its intersection with Z-Z is called the neutral axis.

The development lines of A'B' and C'D' intersect at a point 0 at an angle of θ radians and the radius of E_1F_1 = R.

Let y be the distance(E'G') of any layer H_1G_1 originally parallel to EF.

Then $H_1G_1/ E_1F_1 =(R+y)\theta /R \theta = (R+y)/R$

and the strain at layer H_1G_1= = $(H_1G_1'- HG) / HG = (H_1G_1- HG) / EF$

$$= [(R+y)\theta - R \theta] /R \theta$$

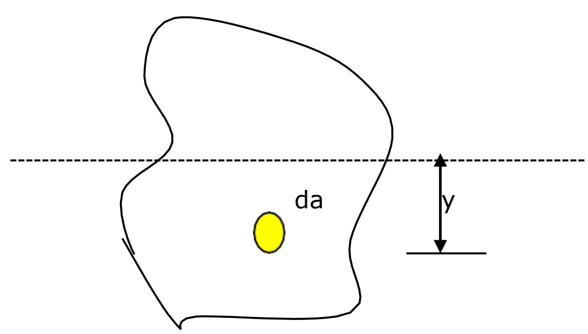
$$= y /R.$$

The relation between stress and strain is σ= E. Therefore

$$\sigma = E. = E. y /R$$

$$\sigma / E = y / R$$

Let us consider an elemental area 'da 'at a distance y, from the Neutral Axis.



Section Modulus(Z)

$$F = \frac{M}{I} . y$$

$$\Rightarrow f_{max} = \frac{M_{max}}{I} . y_{max}$$

$$\text{i,e., } M_{max} = f_{max} . \frac{I}{y_{max}}$$

$$\text{Therefore,} \quad M_{max} = f_{max} . Z$$

Section modulus of a beam is the ratio of moment of inertia of the cross section of the bea m about the neutral axis to the distance of the farthest fiber from neutral axis.

$$\text{Therefore,} \quad Z = \frac{I}{y_{max}} \qquad \text{unit} = mm^3$$

More the section modulus more will be the moment of resistive (or) moment carrying capacity of the beam. For the strongest beam, the section modulus must be maximum.

3.10 Problems

1. **A steel bar 10 cm wide and 8 mm thick is subjected to bending mome nt. The radius of neutral surface is 100 cm. Determine maximum and minimum bending stress in the beam.**

Solution : Assume for steel bar $E = 2 \times 10^5$ N/mm^2

y_{max} = 4mm
R = 1000mm
$f_{max} = E.y_{max}/R = (2 \times 105 \times 4)/1000$

We get maximum bending moment at lower most fiber, Because for a simply supported beam tensile stress (+ve value) is at lower most fiber, while compressive stress is at top most fiber (–ve value).

F_{max} **= 800 N/mm^2**
f_{min} occurs at a distance of – 4mm
R = 1000mm
$f_{min} = E.y_{min}/R = (2 \times 105\ x - 4$
$)/1000$ f_{min}**= –800 N/mm^2**

2. **A simply supported rectangular beam with symmetrical section 200mm in dept h has mome nt of inertia of 2.26 x 10^{-5} m^4 about its neutral axis. Determine the longest span over which the beam would carry a uniformly distributed load of 4kN/m run such that the stress due to bending does not exceed 125 MN/m^2.**

Solution: Given data:

Depth *d* = 200mm = 0.2m

I = Moment of inertia = 2.26 × 10-5 m4

UDL = 4kN/m

Bending stress s = 125 MN/m^2 = 125 × 10^6 N/m^2

Span = ?

Since we know that Maximum bending moment for a simply supported beam with UDL on its entire span is given by = $WL^2/8$

i.e; $M = WL^2/8$ -------------(A)

From bending equation $M/I = f/y_{max}$

$y_{max} = d/2 = 0.2/2 = 0.1$m

M = f.*I*/*y*max = [(125 × 106) × (2.26 × 10^{-5})]/ 0.1 = 28250 Nm

Substituting this value in equation (*A*); we get

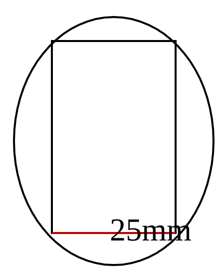
28250 = (4 × 103)$L^2/8$

***L* = 7.52m**

3. **Find the dime nsion of the strongest rectangular beam that can be cut out of a log of 25 mm diameter.**

Solution: $b^2 + d^2 = 25^2$

$d^2 = 25^2 - b^2$



we Know — ; —

M = f (I/y) = f.Z

M will be maximum when Z will be maximum

$Z = I/y = (bd^3/12)/(d/2) = bd^2/6 = b.(25^2 - b^2)/6$

The value of Z maximum at dZ/db = 0;

i.e.; $d/db[25^2b/6 - b^3/6] = 0$

$25^2/6 - 3b^2/6 = 0$

$b^2 = 25^2/3$

b = 14.43 mm

d = 20.41 mm

3.11 Deflection of Beams

3.11.1 INTRODUCTION

Under the action of external loads, the beam is subjected to stresses and deformation at various points along the length. The deformation is caused due to bending moment and shear force. Since the deformation caused due to shear force in shallow beams is very small, it is generally neglected.

3.11.1.1 Elastic Line:

It is a line which represents the deformed shape of the beam. Hence, it is the line along which the longitudinal axis of the beam bends.

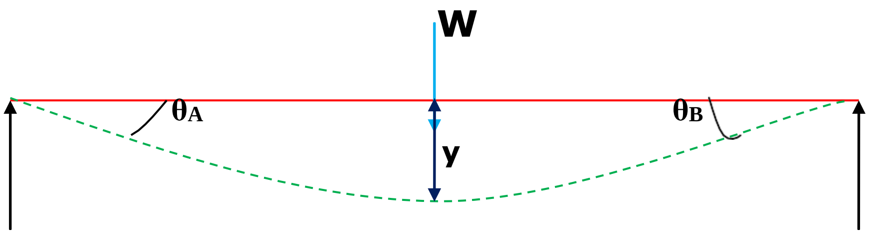
3.11.1.2 Deflection:

Vertical displacement measured from original neutral surface (refer to earlier chapter) to the neutral surface of the deformed beam.

3.11.1.3 Slope:

Angle made by the tangent to the elastic curve with respect to horizontal

The designers have to decide the dimensions of beam not only based on strength requirement but also based on considering deflection. In mechanical components excessive deflection causes mis-alignment and non performance of machine. In building it give rise to psychological unrest and sometimes cracks in roofing materials. Deflection calculations are required to impose consistency conditions in the analysis of indeterminate structures.



3.11.1.4 Strength:

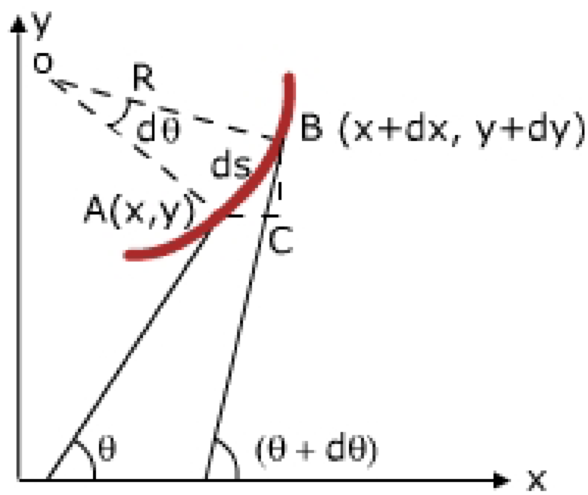
It is a measure of the resistance offered by the beam to load

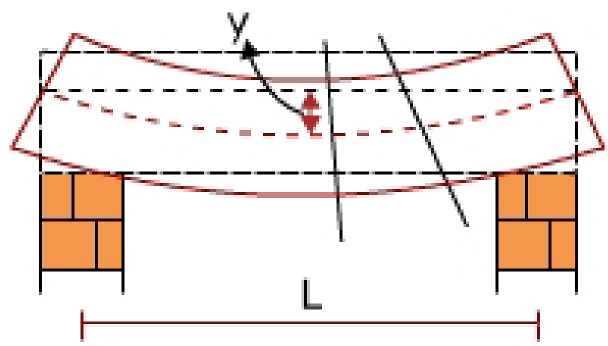
3.11.1.5 Stiffness:

It is a measure at the resistance offered by the beam to deformation. Usually span / deflection is used to denote the stiffness. Greater the stiffness, smaller will be the deflection. The term (EI) called "flexural rigidity" and is used to denote the stiffness.

3.11.2 Flexural Rigidity

The product of Young's modulus and moment of inertia (EI) is used to denote the flexural rigidity.





Let AB be the part of the beam which is bent into an arc of the circle. Let (x,y) be co- ordinates of A and (x + dx, y + dy) be the co-ordinates of B. Let the length of arc AB = ds. Let the tangents at A and B make angles q and (q + d q) with respect to x-axis.

We have __

Differentiating both sides with respect of x;

$$\sec^2\theta \cdot \frac{d\theta}{dx} = \frac{d^2y}{dx^2}$$

$$\sec^2\theta \quad \frac{d\theta}{ds} \quad \frac{ds}{dx} = \frac{d^2y}{dx^2} \qquad \text{----- (1)}$$

we have from figure $ds = Rd\theta$; $\frac{d\theta}{ds} = \frac{1}{R}$

again in Δ^{le} ABC, $\frac{ds}{dx} = \sec\theta$

From eq. 1; $\frac{d^2y}{dx^2} = \sec^2\theta \frac{1}{R} \sec\theta$

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\sec^2\theta \sec\theta} = \frac{\frac{d^2y}{dx^2}}{(1+\tan^2\theta)^{\frac{3}{2}}}$$

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}$$

Since dy/dx is small, its square is still small, neglecting $(dy/dx)^2$; we have

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

From bending theory $\frac{M}{I} = \frac{E}{R}$

$$\frac{M}{EI} = \frac{1}{R} \qquad \text{or}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$M = EI \frac{d^2y}{dx^2}$$

This is also known as Euler - Bernoulli's equation.

NOTE:

- While deriving Y-axis is taken upwards
- Curvature is concave towards the positive y axis.
- This occurs for sagging BM, which is positive.

Sign Convention

Bending moment 

Sagging +vc

If Y is $+^{ve}$ - Deflection is upwards

Y is $-^{ve}$ - Deflection is downwards

If θ is $+^{ve}$ – Slope is Anticlockwise

θ is $-^{ve}$ – Slope is clockwise

Methods of Calculating Deflection and Slope

- Double Integration method
- Macaulay's method
- Strain energy method
- Moment area method
- Conjugate Beam method

Each method has certain advantages and disadvantages.

Relationship between Loading, S.F, BM, Slope and Deflection

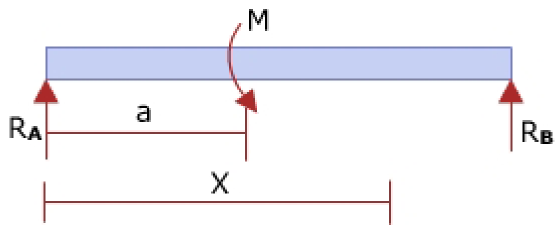
If	Y	-	deflection
Differentiating	$\frac{dy}{dx}$	-	Slope (θ)
Differentiating	$\frac{d^2y}{dx^2}$	-	M. Bending moment
Differentiating	$\frac{dM}{dx}$	=	$\frac{d^3y}{dx^3}$ = Shear force (F)
Differentiating	$\frac{dF}{dx}$	=	$\frac{d^4y}{dx^4}$ = Loading (W)

3.11.3 Macaulay's Method

1. Take the origin on the extreme left.
2. Take a section in the last segment of the beam and calculate BM by considering left portion.
3. Integrate (x-a) using the formula

$$\int (x-a)\,dx = \frac{(x-a)^2}{2}$$

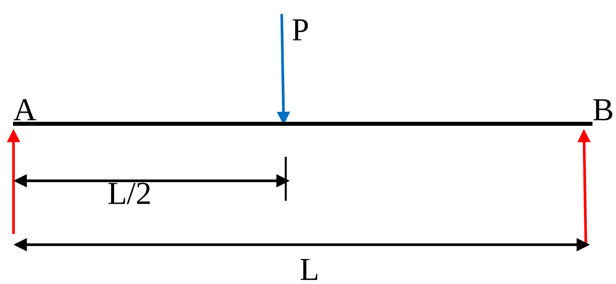
4. If the expression $(x-a)^n$ becomes negative on substituting the value of x, neglect the terms containing the factor $(x-a)^n$
5. If the beam carries UDL and if the section doesn't cuts the UDL, extend the UDL upto the section and impose a UDL in the opposite direction to counteract it.
6. If a couple is acting, the BM equation is modified as; $M = R_A x + M (x-a)^0$.



7. The constant C_1 and C_2 all determined using boundary conditions.
a) S.S. Beam – Deflection is zero at supports
b) Cantilever – Deflection and slope are zero at support.

3.11.4 Problems:

1. **Determine the maximum deflection in a simply supported beam of length L carrying a concentrated load P at its midspan**.



$$EI\,y'' = \tfrac{1}{2}Px - P\langle x - \tfrac{1}{2}L\rangle$$

$EI\,y' = \frac{1}{4}Px^2 - \frac{1}{2}P\langle x - \frac{1}{2}L\rangle^2 + C_1$…………………(1)

$EI\,y = \frac{1}{12}Px^3 - \frac{1}{6}P\langle x - \frac{1}{2}L\rangle^3 + C_1x + C_2$ …………………(2)

At x =0; y =0 □ C_2 =0
At x = L y =0

$0 = \frac{1}{12}PL^3 - \frac{1}{48}PL^3 + C_1L$
$C_1 = -\frac{1}{16}PL^2$
Maximum deflection occurs at x = L/2
Substituting the values of x and C_1 in equation…. (2)

$$EI\,y_{max} = \frac{1}{12}P(\frac{1}{2}L)^3 - \frac{1}{6}P(\frac{1}{2}L - \frac{1}{2}L)^3 - \frac{1}{16}PL^2(\frac{1}{2}L)$$

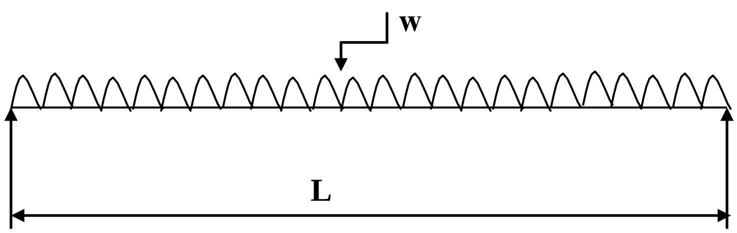
$$y_{max} = -\frac{PL^3}{48EI}$$

The negative sign indicates that the deflection is below the undeformed neural axis

$$\delta_{max} = \frac{PL^3}{48EI}$$

3. Determine the maximum deflection in a simply supported beam of length L carrying a uniformly distributed load ‘w’ for the entire length of the beam.

Solution : From the following fig



$EI\,y'' = \frac{1}{2}w_oLx - \frac{1}{2}w_ox^2$
$EI\,y' = \frac{1}{4}w_oLx^2 - \frac{1}{6}w_ox^3 + C_1$ **………………..(1)**
$EI\,y = \frac{1}{12}w_oLx^3 - \frac{1}{24}w_ox^4 + C_1x + C_2$**…………(2)**

At x =0 y=0 and C_2 =0

At x =L y =0

$$0 = \frac{1}{12} w_o L^4 - \frac{1}{24} w_o L^4 + C_1 L$$

$$C_1 = -\frac{1}{24} w_o L^3$$

Substituting the C_1 values in equation 2 we get

$$EI\, y = \frac{1}{12} w_o L x^3 - \frac{1}{24} w_o x^4 - \frac{1}{24} w_o L^3 x$$

x = L/2, y is maximum due to symmetric loading

$$EI\, y_{max} = \frac{1}{12} w_o L (\frac{1}{2} L)^3 - \frac{1}{24} w_o (\frac{1}{2} L)^4 - \frac{1}{24} w_o L^3 (\frac{1}{2} L)$$

$$EI\, y_{max} = -\frac{5}{384} w_o L^4$$

$$\delta_{max} = \frac{5 w_o L^4}{384 EI}$$

Module 4

TORSION OF SHAFTS

Objectives:

Explain the structural behavior of members subjected to torque, Calculate twist and stress induced in shafts subjected to bending and torsion. & Understand the concept of stability and derive crippling loads for columns

Learning Structure

- 4.1 Bending Moment
- 4.2 ASSUMPTIONS IN TORSION THEORY
- 4.3 Problems
- 4.4 Columns and Struts:
- 4.5 SLENDERNESS RATIO
- 4.6 EFFECTIVE LENGTH OF COLUMN
- .7 Euler’s Theorem
- Outcomes
- Further Reading

4.1 Bending Moment

The moment applied in a vertical plane containing the longitudinal axis is resisted by longitudinal tensile and compressive stresses of varying intensities across the depth of bea m and are called as bending stresses. The moment applied is called Bending Moment.

4.1.1 Torsional Moment

The moment applied in a vertical plane perpendicular to the longitudinal axis i.e., in the plane of the cross section of the member, it causes twisting of layers which will be resisted by the shear stresses. The moment applied is called Torsion Moment or Torsional Moment. Torsion is useful form of transmitting power and its application is seen in screws and shafts.

4.2 ASSUMPTIONS IN TORSION THEORY

1. Material is homogenous and isotropic
2. Plane section remain plane before and after twisting i.e., no warpage of planes.
3. Twist along the shaft is uniform.
4. Radii which are straight before twisting remain straight after twisting.
5. Stresses are within the proportional limit.

4.2.1 DERIVATION OF TORSIONAL EQUATION:

Torsional Rigidity

$$\text{We have} \qquad \theta = \frac{TL}{CI_P}$$

As product (CI_P) is increased deformation q reduces. This product gives the strength of the section to resist torque and is called Torsional rigidity.

Polar Modulus : (Z_P)

$$\text{We have} \qquad \frac{T}{I_p} = \frac{f}{r}$$

Maximum shear stress occurs at surface

$$T = f_s \cdot \frac{I_p}{R}$$

$$T = f_s \cdot Z_p$$

Where Zp is called polar modulus $\qquad Z_p = \frac{I_p}{R}$

POWER TRANSMITTED BY SHAFT

Power transmitted = Torsional moment x Angle through which the torsional moment rotates / unit tank

If the shaft rotates with 'N' rpm

$$= T\left(\frac{N \cdot 2\pi}{60}\right)$$

$$\text{Power transmitted} = \frac{2\pi NT}{60} \ \text{N.m / sec}$$

$$\text{Power transmitted in kw} = \frac{2\pi NT}{60 \times 1000} = \frac{\pi NT}{30{,}000}$$

Note:

N is in rpm and T is in N-m

4.3 Problems:

1. **Find the maximum shear stress induced in a solid circular shaft of diameter 200 mm when the shaft transmits 190 kW power at 200 rpm**

Given data: Power transmitted, P = 190 kW, $I_p = 1.57 \times 10^8$ mm^4

speed N = 200 rpm and diameter of shaft = 200 mm.

Substituting all the values $f_s = 5.78 N/mm^2$.

2. **A solid shaft of mild steel 200 mm in diameter is to be replaced by hollow shaft of allowable shear stress is 22% greater. If the power to be transmitted is to be increased by 20% and the speed of rotation increased by 6%, determine the maximum internal diameter of the hollow shaft. The external diameter of the hollow shaft is to be 200 mm**.

Solution: Given that:

Diameter of solid shaft	$d = 200$ mm
For hollow shaft diameter,	$d_0 = 200$ mm
Shear stress;	$t_H = 1.22\ t_s$
Power transmitted;	$P_H = 1.20\ P_s$
Speed	$N_H = 1.06\ N_s$

As the power transmitted by hollow shaft

$$P_H = 1.20\ P_s$$

$$(2\pi.N_H.T_H)/60 = (2\pi.N_s.T_s)/60 \times 1.20$$

$$N_H.T_H = 1.20\ N_s.T_s$$

$$1.06\ N_s.T_H = 1.20\ N_sT_s$$

$$1.06/1.20\ T_H = T_s$$

$$1.06/1.20 \times \pi/16\ t_H\ [(d_0)^4 - (d_i)^4/d_0] = \pi/16\ t_s.[d]^3$$

$$1.06/1.20 \times 1.22\ t_s\ [(200)^4 - (d_i)^4/200] = t_s \times [200]^3$$

$$d_i = 104 \text{ mm}$$

3. **A solid shaft is subjected to a maximum torque of 1.5 MN.cm Estimate the diameter for the shaft, if the allowable shearing stress and the twist are limited to 1 kN/cm^2 and 1o respectively for 200 cm length of shaft. Take G = 80 × 10^5 N/cm^2**

Solution: Since we have

$$T/I_p = f_s/r = C.\theta/L$$

$$f_s = T.I_p\ r = 1.5 \times 10^6 / \theta/32.d^4\ .\ d/2$$

$$1 \times 10^3 * 2\pi /1.5 \times 10^6 * 32 = 1/d^3$$

$$d = 19.69 \text{ cm}$$

$$\theta = T.L / C.I_p$$

$$1.5 \times 10^6 * 2\pi / 1.5 \times 10^6 * 32 = 1 / d^3$$

$$d = 19.69 \text{ cm}$$

$$\theta = T.L / C.Ip$$

$$1.5 \times 10^6 * 200\ d/80 * 10^5 * \pi/32\ d^4 = \pi/180$$

$d^3 = 1.5 \times 10^6 * 180 * 200 * 32 / (80 * 10^5 * \pi * \pi)$

$d = 27.97$ cm

4. **A hollow circular shaft of 20 mm thickness transmits 300 kW power at 200 r.p.m. Determine the external diameter of the shaft if the shear strain due to torsion is not to exceed 0.00086. Take modulus of rigidity = 0.8×10^5 N/mm^2.**

Solution: Let d_i = inner diameter of circular shaft

d_0 = outer diameter of circular shaft

Then $d_0 = d_i + 2t$ where t = thickness

$d_0 = d_i + 2 * 20$

$d_0 = d_i + 40$

$d_i = d_0 - 40$

Since we have

Power transmitted = 2π NT/60

$300{,}000 = 2\pi * 200 * T / 60$

→ T = 14323900 N mm

Also, we have $C = f_s/y$

→ $0.8 * 10^5 = f_s / 0.00086$

→ $f_s = 68.8$ N/mm^2

Now $T = \pi/16.\ f_s.(d_0^4 - d_i^4 / d_0)$

$14323900 = f_s /16 * 68.8\ (d_0^4 - (d_0 - 40)^4 / d_0)$

$1060334.6\ d_0 = d_0^4 - (d_0 - 40)^4$

$= (d_0^2 - d_0^2 + 80d_0 - 1600)*(d_0^2 + d_0^2 - 80d_0 + 1600)$

$= (80d_0 - 1600)\ (2d_0^2 - 80d_0 + 1600)$

$= 80\ (d_0 - 2_0) * 2 * (d_0^2 - 40\ d_0 + 800)$

$= 160\ (d_0^3 - 40d_0^2 + 800\ d_0 - 20\ d_0^2 + 800\ d_0 - 16000)$

→ $1060334.6\ d_0 / 160 = d_0^3 - 60d_0^2 + 1600d_0 - 16000$

→ $6627\ d_0 = d_0^3 - 60d_0^2 + 1600\ d_0 - 1600$

→ $d_0^3 - 60d_0^2 + 1600d_0 - 6627\ d_0 - 16000 = 0$

→ $d_0^3 - 60d_0^2 - 5027\ d_0 - 16000 = 0$

Using trial and error method to solve the above equation for d_0, we get $d_0 = 107.5$ mm.

Elastic Stability of Columns

4.4 Columns and Struts:

Columns and struts are structural members subjected to compressive forces. Theses members are often subjected to axial forces, although they may be loaded eccentrically. The lengths of these members are large compared to their lateral dimensions. In general vertical compressive members called columns and inclined compressive members are called struts.

4.4.1 CLASSIFICATION OF COLUMNS:

Columns are generally classified in to three general types. The distinction between types of columns is not well, but a generally accepted measure is based on the slenderness ratio (l_e/r_{min}).

4.4.1 .1 Short Column :

A short column essentially fails by crushing and not by buckling. A column is said to be short, if $l_e/b \leq 15$ or $l_e/r_{min} \leq 50$, where l_e = effective length, b = least lateral dimension and r_{min}= minimum radius of gyration.

4.4.1 .2 Long Column :

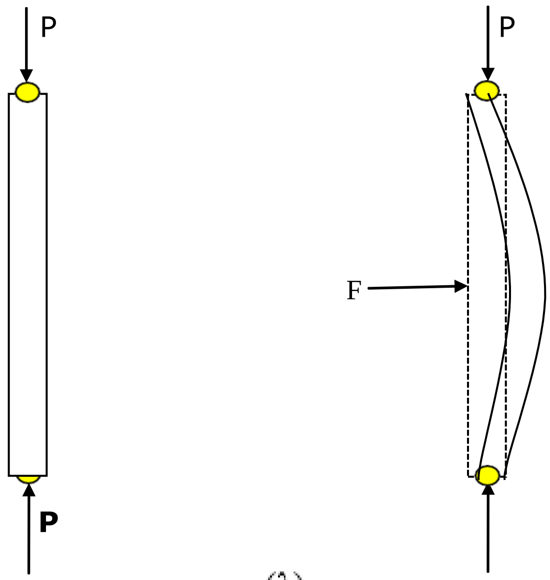
A long column essentially fails by buckling and not by crushing. In long columns, the stress at failure is less than the yield stress. A column is said to be long $l_e/b > 15$ or $l_e/r_{min} > 50$.

4.4.1 .3 Intermediate Column :

An intermediate column is one which fails by a combination of crushing and buckling.

4.4.1.4 Elastic Stability of Column

Consider a long column subjected to an axial load P as shown in figure. The column deflects laterally when a small test load F is applied in lateral direction. If the axial load is small, the column regains its stable position when the test load is removed. At a certain value of the axial load, the column fails to regain its stable position even after the removal of the test load. The column is then said to have failed by buckling and the corresponding axial load is called Critical Load or failure Load or Crippling Load



4.5 SLENDERNESS RATIO (λ)

Slenderness ratio is defined as the ratio of effective length (l_e) of the column to the minimum radius of gyration (r_{min}) of the cross section.

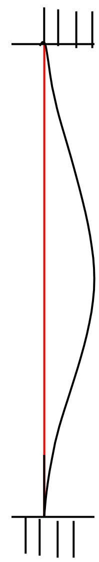
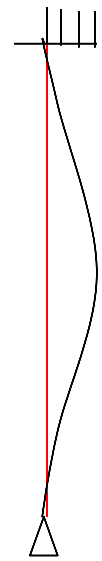
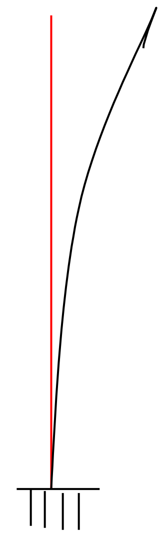
$$\lambda = \frac{l_e}{r_{min}}$$

Since an axially loaded column tends to buckle about the axis of minimum moment of inertia (I_{min}), the minimum radius of gyration is used to calculate slenderness ratio.

Further, —, where A is the cross sectional area of column.

4.6 EFFECTIVE LENGTH OF COLUMN (*l*e)

Effective length is the length of an imaginary column with both ends hinged and whose critical load is the same as the column with given end conditions. It should be noted that the material and geometric properties should be the same in the above columns. The effective length of a column depends on its end condition. Following are the effective lengths for some standard cases.

Both endsare hinged	Both ends arefixed	One end fixed and other end hinged	One end fixed and other end is free
			
Effective Length L_e = L	Effective Length L_e = –	Effective Length L_e = —	Effective Length L_e = 2L

4.7 Euler's Theorem

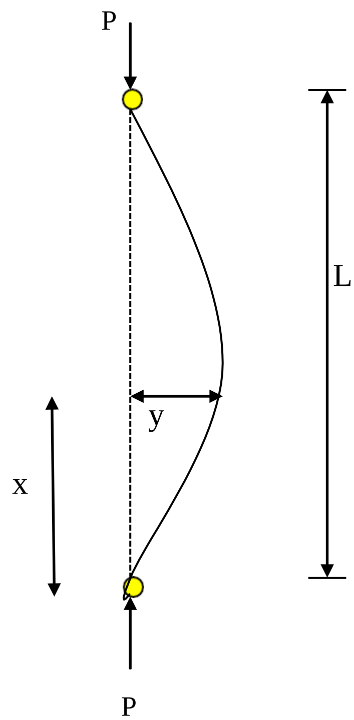
Theoretical analysis of the critical load for long columns was made by the great Swiss mathematician Leonard Euler (pronounced as Oiler). The assumptions made in the analysis are as follows:

- The column is long and fails by buckling.
- The column is axially loaded.
- The column is perfectly straight and the cross sections are uniform (prismatic).
- The column is initially free from stress.
- The column is perfectly elastic, homogeneous and isotropic.

4.7.1 Eulers Critical Load for Long Columns

Case (1) Both ends hinged

Consider a long column with both ends hinged subjected to critical load P as shown.



Consider a section at a distance x from the origin. Let y be the deflection of the column at this section. Bending moment in terms of load P and deflection y is given by

$$M = -P\,y \qquad \text{------------ (1)}$$

We can also write that for beams/columns the bending moment is proportional to the curvature of the beam, which, for small deflection can be expressed as

or ……………..(2)

where E is the Young's modulus and I is the moment of Inertia.

Substituting eq.(1) in eq.(2)

$$-P\,y = EI\,\frac{d^2y}{dx^2}$$

or $$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right) y = 0$$

This is a second order differential equation, which has a general solution form of

$$y = C_1 \sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right) \qquad \text{------(3)}$$

where C_1 and C_2 are constants. The values of constants can be obtained by applying the boundary conditions:

(i) y = 0 at x = 0. That is, the deflection of the column must be zero at each end since it is pinned at each end. Applying these conditions (putting these values into the eq. (3)) gives us the following results: For y to be zero at x =0, the value of C_2 must be zero (since cos (0) = 1).

(i) Substituting y = 0 at x = L in eq. (3) lead to the following.

$$0 = C_1 \sin\left(L\sqrt{\frac{P}{EI}}\right)$$

While for y to be zero at x = L, then either C_1 must be zero (which leaves us with no equation at all, if C_1 and C_2 are both zero), or

$$\sin\left(L\sqrt{\frac{P}{EI}}\right) = 0$$

which results in the fact that

$$\left(L\sqrt{\frac{P}{EI}}\right) = n\,\pi$$

$$\text{or} \qquad L\sqrt{\frac{P}{EI}} = n\,\pi \qquad \text{where } n = 0,1,2,2\ldots$$

$$\text{or} \qquad P = \frac{n^2\pi^2 EI}{L^2}$$

Taking least significant value of n, i.e. n= 1

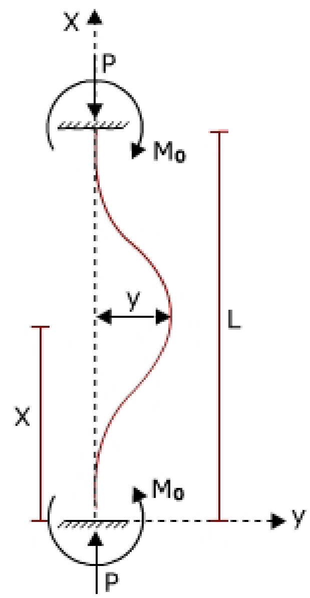
We have $$P = \frac{\pi^2 EI}{L^2}$$

or $$P_E = \frac{\pi^2 EI}{l_e^2}$$

where $l_e = L$.

Case (2) Both ends fixed

Consider a long column with both ends fixed subjected to critical load P as shown.



Consider a section at a distance x from the origin. Let y be the deflection of the column at this section. Bending moment in terms of load P, fixed end moment M 0 and deflection y is given by

$$M = -P\,y + M_0 \qquad \text{-------------(1)}$$

We can also write that for beams/columns the bending moment is proportional to the curvature of the beam, which, for small deflection can be expressed as

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

or $$M = EI\frac{d^2y}{dx^2} \qquad \text{-------------(2)}$$

where E is the Young's modulus and I is the moment of Inertia.

Substituting eq.(1) in eq.(2)

$$-P\,y + M_0 = E\,I\,\frac{d^2y}{dx^2}$$

or $$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = \frac{M_0}{EI}$$

This is a second order differential equation, which has a general solution form of

$$y = C_1 \sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \quad \text{---------} \ (3)$$

where C_1 and C_2 are constants. The values of constants can be obtained by applying the boundary conditions:

(i) y = 0 at x = 0. That is, the deflection of the column must be zero at near end since it is fixed. Applying this condition (putting these values into the eq. (3)) gives us the following result:

$$C_2 = -\frac{M_0}{P}$$

ii) At X = 0 =0, that is, the slope of the column must be zero, since it is fixed.

$$\frac{dy}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(x\sqrt{\frac{P}{EI}}\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(x\sqrt{\frac{P}{EI}}\right) \quad \text{-------}(4)$$

Substituting the boundary condition in eq. (4)

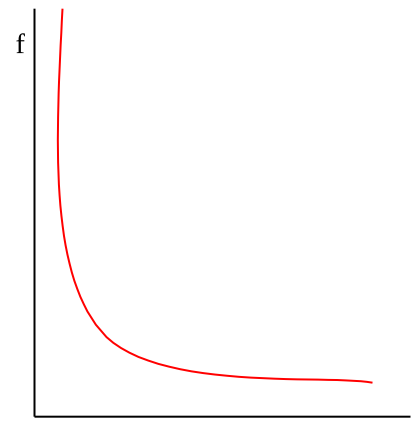
$$0 = C_1\sqrt{\frac{P}{EI}}$$

Hence, $C_1 = 0$

Substituting the constants C_1 and C_2 in eq. (3) leads to the following

$$y = -\frac{M_0}{P}\cos\left(x\sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \qquad \text{---------(5)}$$

The variation of limiting stress 'f' versus slenderness ratio in the above equation is shown below.



The above plot shows that the limiting stress 'f' decreases as increases. In fact, when very small, limiting stress is is close to infinity, which is not rational. Limiting stress cannot be greater than the yield stress of the material.

1. Eulers formula determines the critical load, not the working load. Suitable factor of safety (which is about 1.7 to 2.5) should be considered to obtain the allowable load.

4.7.2 Rankine's critical Load

$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E} \qquad \ldots\ldots\ldots\ldots \quad (1)$$

Where, P_R = Rankine's critical load

$P_C = f_C\, A$ = Crushing load for short columns

$P_E = \dfrac{\pi^2 EI}{l_e^2}$ = Euler's critical load for long columns

Rankine Gordon Load is given by the following empirical formula,

This relationship is assumed to be valid for short, medium and long columns. This relation can be used to find the load carrying capacity of a column subjected to crushing and/or buckling.

From eq. (1)

Substituting P_C and P_E in the above relation

$$P_R = \frac{f_c\ A}{1 + \left[\dfrac{\dfrac{f_c\ A}{\pi^2 E I}}{l_e^2}\right]} = \frac{f_c\ A}{1 + \left(\dfrac{f_c}{\pi^2 E}\right)\left[\dfrac{l_e^2\ A}{I}\right]}$$

Since $$\frac{I_{min}}{A} = (r_{min})^2$$

$$P_R = \frac{f_c\ A}{1 + a\left[\dfrac{l_e}{r_{min}}\right]^2}$$

Module 5: Theories of Failure

Objectives:

Various types of theories of failure and its importance

Learning Structure

- 5.0 Introduction
- 5.1 Stress-Strain relationships
- 5.2 Types of Failure
- 5.3 Use of factor of safety in design
- 5.4 Theories of Failure
- 5.5 Problems
- Outcomes
- Further reading

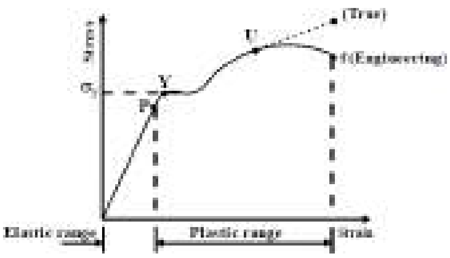
5.0 Introduction:

Failure indicate either fracture or permanent deformation beyond the operational range due to yielding of a member. In the process of designing a machine element or a structural member, precautions has to be taken to avoid failure under service conditions.

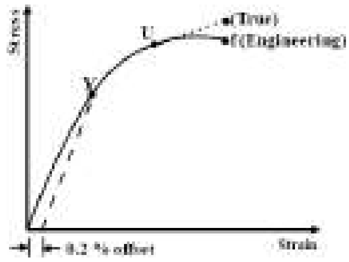
When a member of a structure or a machine element is subjected to a system of complex stress system, prediction of mode of failure is necessary to involve in appropriate design methodology. Theories of failure or also known as failure criteria are developed to aid design.

5.1 Stress-Strain relationships:

Following Figure-1 represents stress-strain relationship for different type of materials.

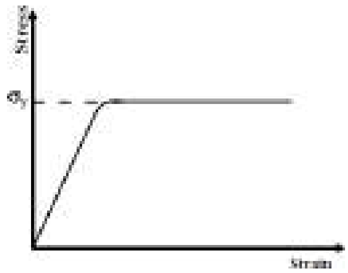


Ductile material e.g. low carbon steel



Low ductility

Brittle material



Elastic – perfectly plastic material

Figure-: Stress-Strain Relationship

Bars of ductile materials subjected to tension show a linear range within which the materials exhibit elastic behaviour whereas for brittle materials yield zone cannot be identified. In general, various materials under similar test conditions reveal different behaviour. The cause of failure of a ductile material need not be same as that of the brittle material.

5.2 Types of Failure:

The two types of failure are,

Yielding - This is due to excessive inelastic deformation rendering the structural member or machine part unsuitable to perform its function. This mostly occurs in ductile materials.

Fracture - In this case, the member or component tears apart in two or more parts. This mostly occurs in brittle materials.

5.3 Use of factor of safety in design:

In designing a member to carry a given load without failure, usually a factor of safety (FS or N) is used. The purpose is to design the member in such a way that it can carry N times the actual working load without failure. Factor of safety is defined as Factor of Safety (FS) = Ultimate Stress/Allowable Stress.

5.4 Theories of Failure:

a) Maximum Principal Stress Theory (Rankine Theory)
b) Maximum Principal Strain Theory (St. Venant's theory)
c) Maximum Shear Stress Theory (Tresca theory)
d) Maximum Strain Energy Theory (Beltrami's theory)

5.4.1 Maximum Principal Stress Theory (Rankine theory)

According to this, if one of the principal stresses σ_1 (maximum principal stress), σ_2 (minimum principal stress) or σ_3 exceeds the yield stress (σ_y), yielding would occur. In a two dimensional loading situation for a ductile material where tensile and compressive yield stress are nearly of same magnitude

$\sigma_1 = \pm \sigma_y$ $\sigma_2 = \pm\sigma_y$

Yield surface for the situation is, as shown in Figure-2

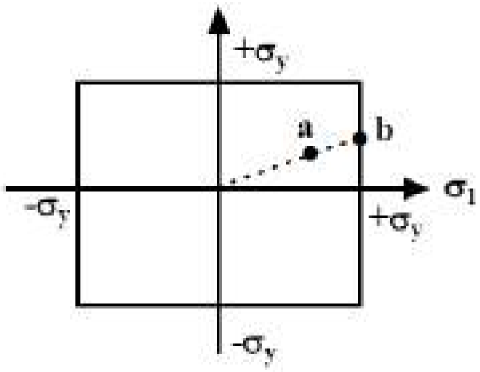


Figure- 2: Yield surface corresponding to maximum principal stress theory

Yielding occurs when the state of stress is at the boundary of the rectangle. Consider, for example, the state of stress of a thin walled pressure vessel. Here $\sigma_1 = 2\sigma_2$, σ_1 being the circumferential or hoop stress and σ_2 the axial stress. As the pressure in the vessel increases, the stress follows the dotted line. At a point (say) a, the stresses are still within the elastic limit but at b, σ_1 reaches σ_y although σ_2 is still less than σ_y. Yielding will then begin at point b. This theory of yielding has very poor agreement with experiment. However, this theory is being used successfully for brittle materials.

5.4.2 Maximum Principal Strain Theory (St. Venant's Theory)

According to this theory, yielding will occur when the maximum principal strain just exceeds the strain at the tensile yield point in either simple tension or compression. If ε_1 and ε_2 are maximum and minimum principal strains corresponding to σ_1 and σ_2, in the limiting case

$\varepsilon_1 = (1/E)(\sigma_1 - \nu\sigma_2)$ $|\sigma_1| \geq |\sigma_2|$

$\varepsilon_2 = (1/E)(\sigma_2 - \nu\sigma_1)$ $|\sigma_2| \geq |\sigma_1|$

This results in,

$E\,\varepsilon_1 = \sigma_1 - \nu\sigma_2 = \pm\,\sigma_0$
$E\,\varepsilon_2 = \sigma_2 - \nu\sigma_1 = \pm\,\sigma_0$

The boundary of a yield surface in this case is shown in Figure – 3.

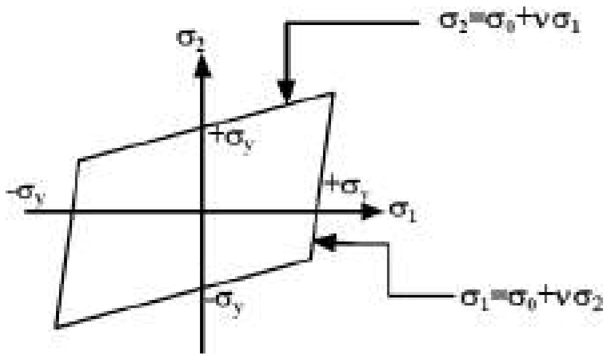


Figure-3: Yield surface corresponding to maximum principal strain theory

5.4.3 Maximum Shear Stress Theory (Tresca theory)

According to this theory, yielding would occur when the maximum shear stress just exceeds the shear stress at the tensile yield point. At the tensile yield point $\sigma_2 = \sigma_3 = 0$ and thus maximum shear stress is $\sigma_y/2$. This gives us six conditions for a three-dimensional stress situation:

$\sigma_1 - \sigma_2 = \pm\,\sigma_y$
$\sigma_2 - \sigma_3 = \pm\,\sigma_y$
$\sigma_3 - \sigma_1 = \pm\,\sigma_y$

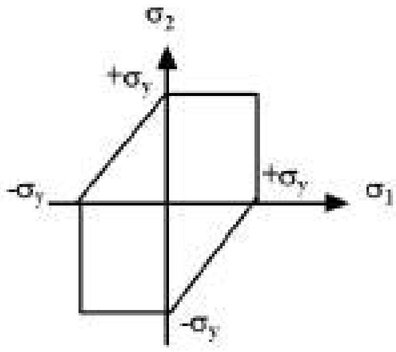


Figure – 4: Yield surface corresponding to maximum shear stress theory

In a biaxial stress situation (Figure - 4) case, $\sigma_3 = 0$ and this gives

$\sigma_1 - \sigma_2 = \sigma_y$	if $\sigma_1 > 0, \sigma_2 < 0$
$\sigma_1 - \sigma_2 = -\sigma_y$	if $\sigma_1 < 0, \sigma_2 > 0$
$\sigma_2 = \sigma_y$	if $\sigma_2 > \sigma_1 > 0$
$\sigma_1 = -\sigma_y$	if $\sigma_1 < \sigma_2 < 0$
$\sigma_1 = -\sigma_y$	if $\sigma_1 > \sigma_2 > 0$
$\sigma_2 = -\sigma_y$	if $\sigma_2 < \sigma_1 < 0$

This criterion agrees well with experiment.

In the case of pure shear, $\sigma_1 = - \sigma_2 = k$ (say), $\sigma_3 = 0$
and this gives $\sigma_1 - \sigma_2 = 2k = \sigma_y$
This indicates that yield stress in pure shear is half the tensile yield stress and this is also seen in the Mohr's circle (Figure - 5) for pure shear.

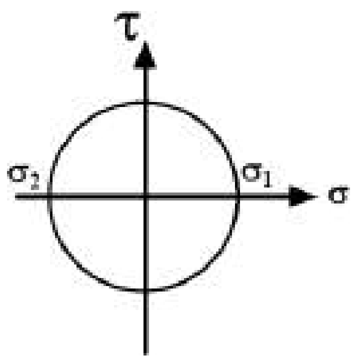


Figure – 5: Mohr's circle for

pure shear

5.4.4 Maximum strain energy theory (Beltrami's theory)

According to this theory failure would occur when the total strain energy absorbed at a point per unit volume exceeds the strain energy absorbed per unit volume at the tensile yield point. This may be expressed as,

$$(1/2)(\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3) = (1/2)\, \sigma_y \varepsilon_y$$

Substituting ε_1, ε_2, ε_3 and ε_y in terms of the stresses we have

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\upsilon\,(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1) = \sigma_y^2$$

$$(\sigma_1/\sigma_y)^2 + (\sigma_2/\sigma_y)^2 - 2\nu(\sigma_1 \sigma_2/\sigma_y^2) = 1$$

The above equation represents an ellipse and the yield surface is shown in F igure - 6

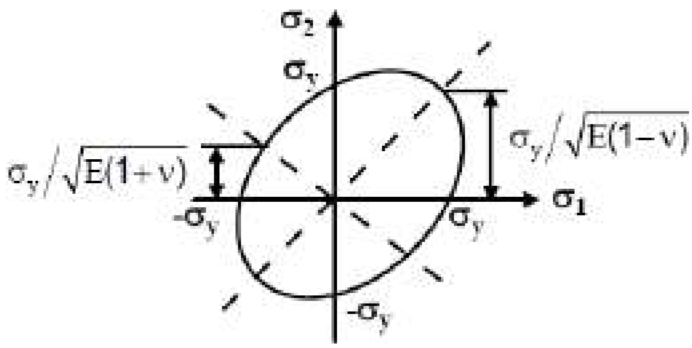


Figure – 6: Yield surface corresponding to Maximum strain energy theory.

It has been shown earlier that only distortion energy can cause yielding but in the above expression at sufficiently high hydrostatic pressure $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ (say), yielding may also occur. From the above we may write $\sigma^2(3 - 2\nu) = \sigma_y^2$ and if $\nu \sim 0.3$, at stress level lower than yield stress, yielding would occur. This is in contrast to the experimental as well as analytical conclusion and the theory is not appropriate.

5.4.5 Superposition of yield surfaces of different failure theories:

A comparison among the different failure theories can be made by superposing the yield surfaces as shown in figure – 7. It is clear that an immediate assessment of failure probability can be made just by plotting any experimental in the combined yield surface. Failure of ductile materials is most accurately governed by the distortion energy theory where as the maximum principal strain theory is used for brittle materials.

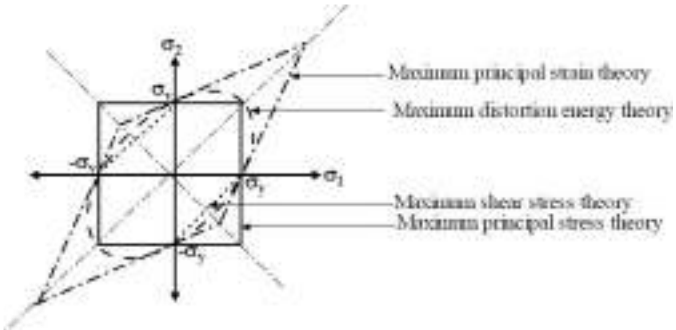


Figure – 7: Comparison of different failure theories

5.5 Problems:

Numerical-1: A shaft is loaded by a torque of 5 KN-m. The material has a yield point of 350 MPa. Find the required diameter using Maximum shear stress theory. Take a factor of safety of 2.5.

Torsional Shear Stress, $\tau = 16T/\pi d^3$, where d represents diameter of the shaft

Maximum Shear Stress theory, √

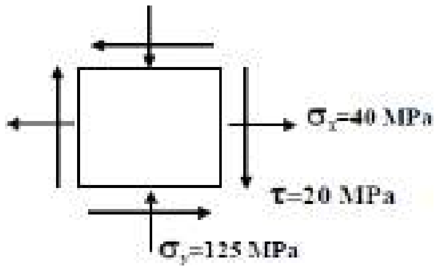
Factor of Safety (FS) = Ultimate Stress/Allowable Stress

Since $\sigma_x = \sigma_y = 0$, $\tau_{max} = 25.46 \text{ X } 10^3/d^3$

Therefore $25.46 \text{ X } 10^3/d^3 = \sigma_y/(2*FS) = 350*10^6/(2*2.5)$

Hence, d = 71.3 mm

Numerical-2: The state of stress at a point for a material is shown in the following figure Find the factor of safety using (a) Maximum shear stress theory Take the tensile yield strength of the material as 400 MPa.



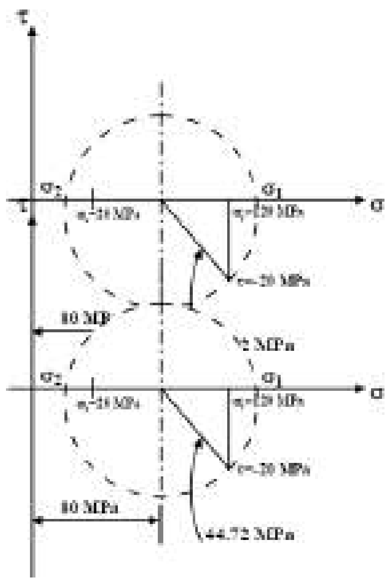
From the Mohr's circle shown below we determine,

σ_1 = 42.38MPa and
σ_2 = -127.38MPa

from Maximum Shear Stress theory

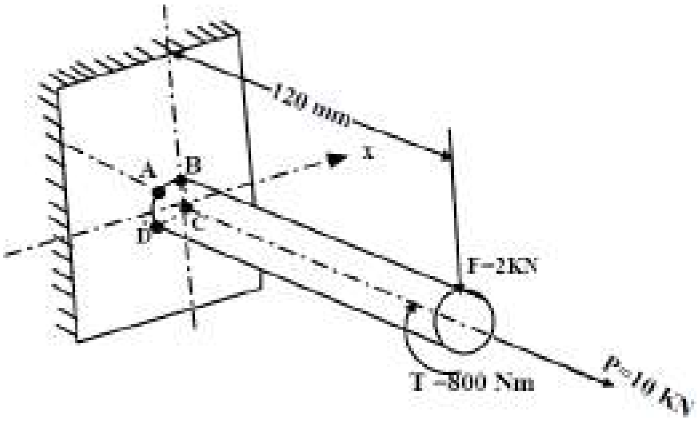
$(\sigma_1 - \sigma_2)/2 = \sigma_y/(2*FS)$

By substitution and calculation factor of safety FS = 2.356

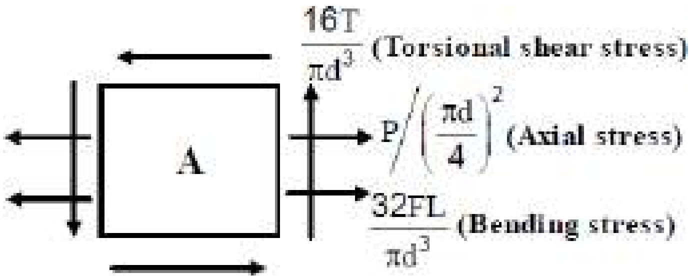


Numerical-3: A cantilever rod is loaded as shown in the following figure. If the tensile yield strength of the material is 300 MPa determine the

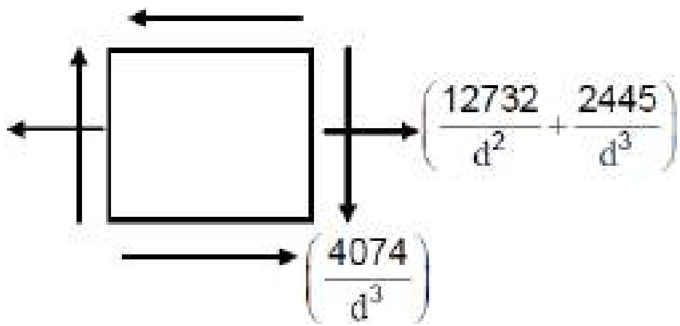
rod diameter using (a) Maximum principal stress theory (b) Maximum shear stress theory



At the outset it is necessary to identify the mostly stressed element. Torsional shear stress as well as axial normal stress is the same throughout the length of the rod but the bearing stress is largest at the welded end. Now among the four corner elements on the rod, the element A is mostly loaded as shown in following **figure**



Shear stress due to bending VQ/It is also developed but this is neglected due to its small value compared to the other stresses. Substituting values of T, P, F and L, the elemental stresses may be shown as in following **figure.**



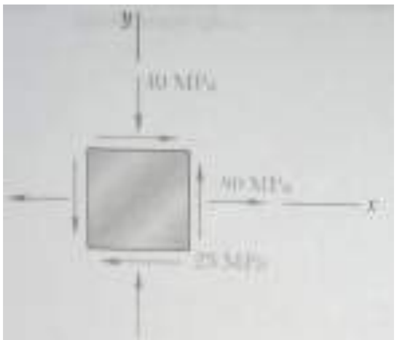
The principal stress for the case is determined by the following equation,

$$\sigma_{1,2} = \frac{1}{2}\left(\frac{12732}{d^2} + \frac{2445}{d^3}\right) \pm \sqrt{\frac{1}{4}\left(\frac{12732}{d^2} + \frac{2445}{d^3}\right)^2 + \left(\frac{4074}{d^3}\right)^2}$$

By Maximum Principal Stress Theory, Setting, $\sigma_1 = \sigma_y$ we get d = 26.67mm

By maximum shear stress theory by setting $(\sigma_1 - \sigma_2)/2 = \sigma_y/2$, we get, d = 30.63mm

Numerical-4: The state of plane stress shown occurs at a critical point of a steel machine component. As a result of several tensile tests it has been found that the tensile yield strength is σ_y=250MPa for the grade of steel used. Determine the factor of safety with respect to yield using maximum shearing stress criterion.

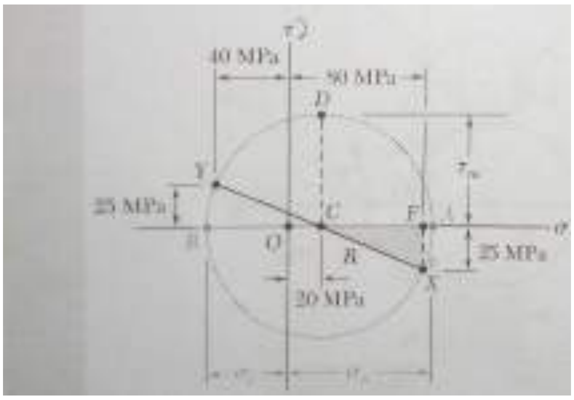


Construction of the Mohr's circle determines

σ_{avg} = ½ (80-40) = 20MPa and $\tau_m = (60^2+25^2)^{1/2}$ = 65MPa
σ_a= 20+65 = 85 MPa and σ_b= 20-65 = -45 MPa

The corresponding shearing stress at yield is τ_y= ½ σ_y = ½ (250) = 125MPa

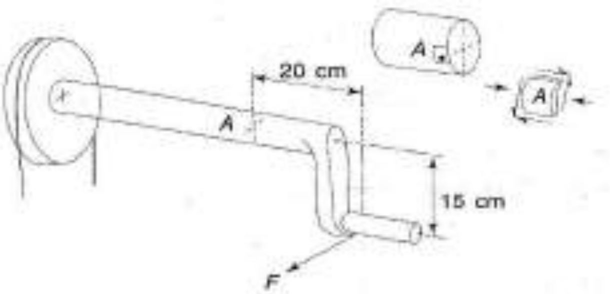
Factor of safety, FS = τ_m / τ_y = 125/65 = 1.92



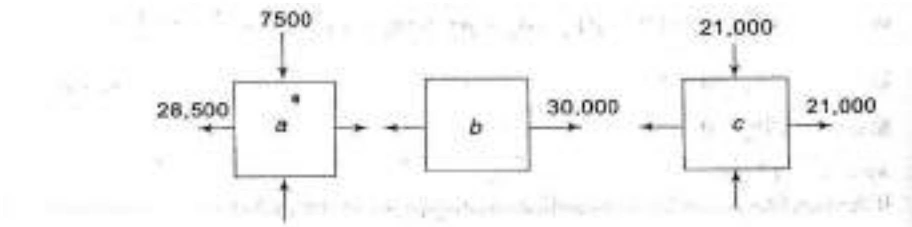
Summary:
Different types of loading and criterion for design of structural members/machine parts subjected to static loading based on different failure theories have been discussed. Development of yield surface and optimization of design criterion for ductile and brittle materials were illustrated.

<u>Assignments:</u>

Assignment-1: A Force F = 45,000N is necessary to rotate the shaft shown in the following figure at uniform speed. The crank shaft is made of ductile steel whose elastic limit is 207,000 kPa, both in tension and compression. With E = 207 X 10^6 kPa and ν = 0.25, determine the diameter of the shaft using maximum shear stress theory, using factor of safety = 2. Consider a point on the periphery at section A for analysis **(Answer, d = 10.4 cm)**



Assignment-2: Following figure shows three elements a, b and c subjected to different states of stress. Which one of these three, do you think will yield first according to i) maximum stress theory, ii) maximum strain theory, and iii) maximum shear stress theory? Assume Poisson's ratio ν = 0.25 **[Answer: i) b, ii) a, and iii) c]**



Assignment-3: Determine the diameter of a ductile steel bar if the tensile load F is 35,000N and the torsional moment T is 1800N.m. Use factor of safety = 1.5. E = 207*10^6kPa and σ_{yp} = 207,000kPa. Use the maximum shear stress theory. **(Answer: d = 4.1cm)**

Assignment-4: At a pint in a steel member, the state of stress shown in Figure. The tensile elastic limit is 413.7kPa. If the shearing stress at a point is 206.85kPa, when yielding starts, what is the tensile stress σ at the point according to maximum shearing stress theory? **(Answer: Zero)**